MULTI-CRITERIA STUDY OF THE QUALITY INDICATORS OF QUARTZ DRIVING MASSES BY FRACTIONAL-RATIONAL GENERALIZED FUNCTIONS

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ABSTRACT

Regression mathematical models of the second full order were obtained for six quality indicators (objective parameters) for driving masses type "Fosuk". The adequacy of the obtained regression equations for the quality indicators was approved. The optimal values of amount of clay substance Al_2O_3 , %, and temperature of heat treatment, °C, were determined. The lines of constant values for each of the quality indicators in the range of variation in amount of clay substance Al_2O_3 and heat treatment temperature are presented. A multi-criteria optimization based on arithmetic mean, geometric mean and harmonic mean functions of usefulness was performed. An analysis of strategies for multi-criteria decision-making using a fractional-rational generalized function of usefulness is carried out.

<u>Keywords</u>: Fosuk type driving masses, clay substance, temperature of heat treatment, open porosity, compressive strength, temperature of deformation, regression model, function of usefulness, fractional-rational function.

INTRODUCTION

For the construction of the internal refractory ceramic linings of the steel foundry ladles in the majority of steel productions, a semi-acid driving masses of phosphate binding type "Fosuk" are used, the one-component composition of which is based on natural kaolin. The driving masses consist of a refractory filler and one or more binding substances, according to reference data, the most commonly used ingredients are [1]:

- Silicon oxide compositions (quartzite, ganister materials (fillers) and ganister fillers with additives);
- Corundum and aluminosilicate compositions (tabular Al₂O₃, electrocorundum, calcined bauxite, fireclay, andalusite, etc.). Depending on the application, these fillers can be used in binding compounds with silicon carbide and carbon-containing materials;
- Main ingredients (magnesite, dolomite, chromite ore, dichromium trioxide, synthetic spinels, etc.);
- Other ingredients/components (silicon carbide, zirconium silicate, etc.).

The most common method of construction of the refractory linings is by mechanized ramming under the applied external pressure, usually by pneumatic or electric machines [1, 2]. Since there is a shortage of wetting fluid in the driving masses, tamping must be achieved by mechanical force. This requires a higher content of intermediate fractions. Air-curing and thermal-curing bindings based on silicates are widely used [1]. The most commonly bindings used in driving masses are:

- Inorganic binding substances: clay, phosphoric acid and phosphates, sulfates, silicates, colloidal Al_2O_3 and SiO_2 , boric acid and borates, etc.
- Organic binding substances: carboxyl-methyl-cellulose, lignosulfonates, molasses, synthetic resins, etc.

In his work Serbezov determine and discuss the optimal ranges (limits) of clay substance content in the driving masses and the process temperatures to which various parts of refractory linings [2]. It was found that the content of clay substance below 18 % leads to a significant decrease in the binding ability and monolithicity of the driving masses. With a content of clay substance above 30 %, it leads to a decrease in the refractory indicators of the driving masses.

In the present study, the main ceramic characteristics of quartz driving masses consisting of a clay substance

(Al₂O₃) and introduced technical phosphoric acid (H₃PO₄) were studied in order to ensure sufficient mechanical strength and density of the mass in a certain operating temperature interval [2].

The present study includes:

- Regression models of the second full order for the quality indicators obtaining, based on the results of a planned experiment and proving the adequacy of the regression models;
- Optimization of the considered quality indicators and comparative analysis of the optimal solutions;
- Multi-criteria optimization by generalized fractionalrational function of usefulness.

The following quality indicators (objective parameter) of the of "Fosuk" type driving masses were studied:

Y₁ - Linear changes, %, (minimum is required);

Y₂ - Open porosity, %, (minimum is required);

Y₂ - Compressive strength, MPa, (maximum is required);

Y₄ - Beginning of deformation, °C, (maximum is

required);

Y₅ - 4 % deformation, °C, (maximum is required);

Y₆ - 40 % deformation, °C, (maximum is required).

The independent variables (process control parameters) are:

X₁ - amount of clay substance, Al₂O₂, %

 X_2 - temperature of heat treatment, °C

EXPERIMENTAL

Experimental design and results for the driving masses "Fosuk"

Adequate experimentally-statistically mathematical models of the second full ordere were obtained to determine the effect of the independent variables, based on data from a experimental design using an optimal composition plan of the experiments [2]. The optimal compositional experimental design and the results are given in Table 1.

Table	ble 1. Experimental results for driving masses "Fosuk".									
No	X ₁	X_2	Y	Y ₂	Y ₃	Y ₄	Y_5	Y ₆		
-	Al ₂ O ₃ , %	°C	%	%	MPa	°C	°C	°C		
1	4.8	200	0.39	27.24	10.0	1460	1470	1490		
2	4.8	400	0.53	28.97	12.0	1450	1480	1490		
3	4.8	600	0.71	29.20	10.8	1440	1480	1480		
4	4.8	800	1.20	30.60	7.7	1420	1460	1480		
5	4.8	1000	1.25	30.72	8.2	1410	1460	1480		
6	4.8	1200	1.38	30.00	8.8	1450	1480	1510		
7	4.8	1400	2.26	30.53	14.0	1460	1500	1530		
8	4.8	1600	2.25	24.89	39.7	1510	1530	1550		
9	6.34	200	0.27	26.21	10.8	1340	1400	1420		
10	6.34	400	0.34	27.43	12.8	1340	1400	1430		
11	6.34	600	0.39	28.45	12.2	1330	1390	1420		
12	6.34	800	0.90	28.71	8.6	1320	1390	1420		
13	6.34	1000	1.05	29.98	9.4	1320	1400	1440		
14	6.34	1200	1.20	29.28	11.0	1370	1420	1460		
15	6.34	1400	1.70	27.83	16.5	1420	1470	1500		
16	6.34	1600	1.99	20.00	50.0	1470	1510	1530		
17	7.2	200	0.14	24.33	11.5	1320	1400	1420		
18	7.2	400	0.16	24.81	13.2	1330	1400	1420		
19	7.2	600	0.18	26.39	12.4	1320	1390	1410		
20	7.2	800	0.74	28.03	11.7	1310	1380	1410		
21	7.2	1000	0.88	28.41	11.9	1310	1400	1430		
22	7.2	1200	1.10	27.89	12.1	1360	1410	1450		
23	7.2	1400	1.48	25.30	29.2	1390	1440	1480		
24	7.2	1600	1.45	19.80	45.4	1480	1500	1520		

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Table /	Calculated	Values of	the co	netti ciente	in th	e regression	eallaftons
raute 2.	Carculated	values of			III UII	o regression	equations.

	Line	ar changes,		-8	Open porosity, (Y ₂)					
b _{ij}	Regression coefficient	F _{table}	F _{calc}	R	Regression coefficient	F _{table}	F _{calc}	R		
b_0	-0.48386103				16.694089					
b,	0.28738394				2.6822771	1				
b,	0.00138948	2 ==2		0.050	0.02283558	2.552	15.005	0.000		
$b_{1,1}$	-0.03152371	2.773	81.547	0.979	-0.30078872	2.773	17.205	0.909		
b _{2,2}	0.00000034				-0.00001159	1				
b _{1,2}	-0.00012122				-0.00054329					
1,2	Compre	ssive streng	$gth, (Y_3)$		Beginning	g of deform	ation, (Y ₄)			
b_{ij}	Regression	E	E	R	Regression	F	F	R		
,	coefficient	table	F _{table} F _{calc}		coefficient	F _{table}	F _{calc}	IX		
b_0	38.135685			0.899	2457.6731	2.773	89.347	0.980		
b_1	-4.0280964		15.151		-281.44682					
b_2	-0.06950096	2.773			-0.3955005					
b _{1,1}	0.29525949	2.773	13.131		17.885458					
b _{2,2}	0.0000387				0.00015823					
b _{1,2}	0.00263378				0.02910703]				
	4% d	eformation	, (Y ₅)		40% deformation, (Y ₆)					
b_{ij}	Regression	Б	E	R	Regression	Б		R		
,	coefficient	F _{table}	F _{calc}	K	coefficient	F _{table}	F _{calc}			
b_0	2142.796				1972.8289]				
b_1	-191.2558				-135.34256					
b_2	-0.22349771	2.773	66.574	0.974	-0.1780997	2.773	161 046	0.989		
b _{1,1}	12.600045	2.113	00.3/4	0.9/4	8.3449738	2.773	161.046	0.707		
b _{2,2}	0.00010913				0.00009276					
b _{1,2}	0.01325584				0.01191523					

Regression models for the quality indicators, Y_j , j = 1, 2, ..., 6

For a statistical analysis of the regression models, applied software product QStatLab [3] was used.

The regression equations of the quality indicators have the following form:

$$\begin{split} Y_j &= b_0 + \sum_{i=1}^m b_i X_i + \sum_{i=1}^{m-1} \sum_{j=i+1}^m b_{ij} X_i X_j + \sum_{i=1}^m b_{ii} X_{ii}^2 + \dots \\ , \, m=2 \end{split} \label{eq:Y_j}$$

The calculated values for the selected coefficients in the regression equations of full second-order are presented in Table 2. The significance of the coefficients [4] was determined at the level of significance $\alpha = 0.05$.

From the values presented in Table 2 it can be seen that all the multiple regression coefficients are high and the calculated values of the Fisher criterion $F_{calc,j}$, $j = 1, 2, \ldots, 6$ significantly exceed the table value F_{table} (0.050, 5,

18) = 2.773, which confirms that the obtained regression models adequately describe the experimental data.

As a illustrative example, a normal plot of residuals, a normal plot of effects and residuals in the order of obtaining for two quality indicators: linear changes (Y_1) and beginning of deformation (Y_4) are presented on Figs. 1 - 4.

From the Figs. 1 - 4 the following more important conclusions can be summarized:

On Fig. 1 it can be seen that all the data fit adequately around the straight line, indicating that the errors of the experimental data are within the experimental tolerances and there are no gross errors. "Probability" is the confidence probability the equation to meet an adequacy of not less than 95 %. The parameter "AD" is the Anderson-Darling parameter for testing normality of distribution. The smaller AD value, the closer the distribution is to normal. The parameter "p" is the

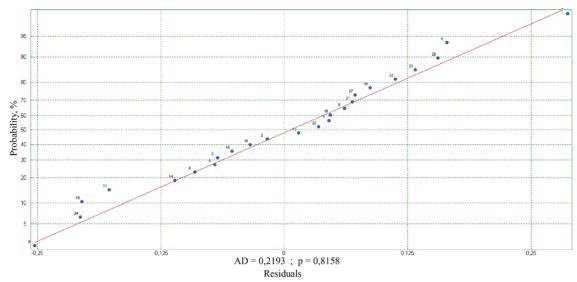


Fig. 1. Normal graph of the residuals for Linear changes (Y_1) .

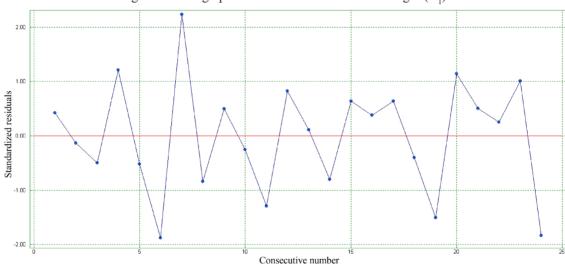


Fig. 2. Residuals in order of obtaining for Linear changes (Y₁).

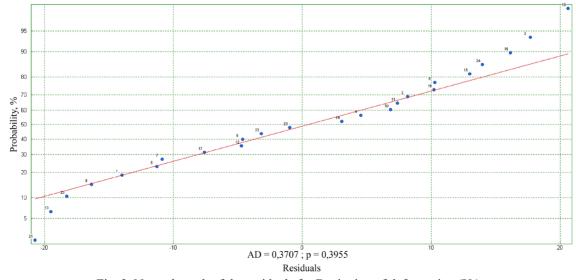


Fig. 3. Normal graph of the residuals for Beginning of deformation (Y_4) .

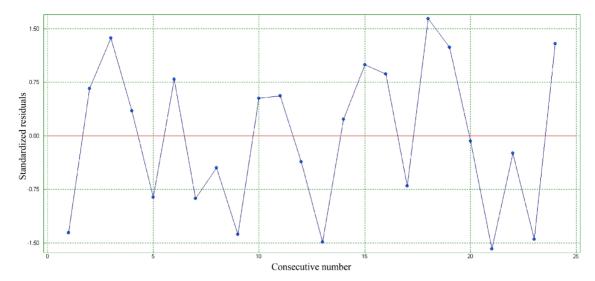


Fig. 4. Residuals in order of obtaining for Beginning of deformation (Y₄).

Table 3. Optimal values for all quality indicators Y_1 до Y_6 .

	Quality indicator	Desired value	X_1^*	X_2^*	Y _i optimum
	Quality indicator	Desired value	% Al ₂ O ₃	°C	j = 1, 2,, 6
Y	Linear changes, %	Minimum	7.2	200	0.14
Y	Open porosity, %	Minimum	7.2	1600	21.02
Y ₃	Compressive strength, MPa	Maximum	7.2	1600	42.651
Y_4	Beginning of deformation, °C	Maximum	4.8	1600	1515
Y_5	4% deformation, °C	Maximum	4.8	1600	1538
Y	40% deformation, °C	Maximum	4.8	1600	1559

Probability that the distribution is normally distributed. If p > 0.1 it can be assumed that the distribution is normal [3].

For the quality indicator "Beginning of deformation", it can be seen that on the graph of the residuals (Fig. 3) 79% of the data are located with small deviations around the straight line, but 5 numbers of the data (No 3, 13, 18, 19 and 21) deviate significantly. The probability of error in the data is very small and the disposition of the points indicates that the data are not satisfactorily described by a second full order regression equation, but the equation can be used for practical purposes because about 80% of the experimental data satisfy Fisher's criterion.

On Fig. 4, it can be seen that the error of the predicted values for "Beginning of deformation" is relatively small: $\pm 1.5 \sigma$ of the "three sigma" rule (with a small exceedance for two experiments). For the quality indicator "Linear Changes" (Fig. 2), this error is within $\pm 1.2 \sigma$ with a small exceedance for one experiment. The "Standardized residues" are standardized over the entire range of variation.

For all six quality indicators, the clay substance has

the greatest influence on them through the coefficients b_1 and b_{11} .

Comparative analysis of the optimal solutions for the quality indicators

A "Genetic Algorithm" [5, 6] was used to find the optimal solution for each quality indicator. The results are presented in Table 3 and illustrated on Figs. 5 - 10.

From Table 3 it can be summarized that all quality indicators from Y_2 to Y_6 obtain the desired optimal values at the same values of the independent variables X_1^* and X_2^* . Only for the linear changes (Y_1) , the independent variable X_2^* strongly differs. This means that for the optimal process control it is necessary to be used a strategy for a compromise optimal solution finding for the independent variables X_1 and X_2

In Fig. 5 to Fig. 10 the lines of constant values for the quality indicators Y_j , j = 1, 2, ..., 6 in the range of changing for clay substance amount Al_2O_3 , %, and temperature of heat treatment, °C, are presented. From these figures the fast value determination for every

one of the quality indicators Yj, j = 1, 2, ..., 6 for a set value of the two independent variables X_1 and X_2 can be determined. Also the optimal range of changing for each quality indicator can be determined. After determining the optimum ranges of each parameter and overlay them within the studied limits for the independent variables (control parameters), the range that will provide the optimum specified parameters can be determined.

Multicriteria optimization by fractional-rational generalized function of usefulness for the six quality indicators

A multicriteria study using fractional-rational generalized functions was done [7]. In order to eliminate the influence of the different dimensions of the quality indicators (technological, physical, economic, etc.), they

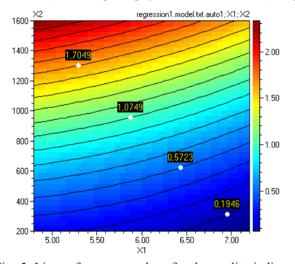


Fig. 5. Lines of constant values for the quality indicator Linear changes (Y_1) .

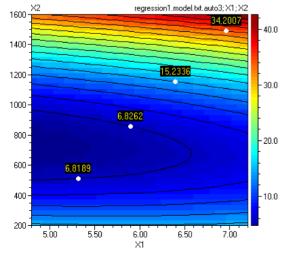


Fig. 7. Lines of constant values for the quality indicator Compressive strength (Y_3) .

are normalized as usefulness in the limits $(0 \div +1)$, taken as absolute values [8]:

$$\eta_{j}(\mathbf{X}) = \left| \frac{Y_{j}(\mathbf{X}) - Y_{j}^{\text{pes}}(\mathbf{X})}{Y_{j,\text{max}}(\mathbf{X}) - Y_{j,\text{min}}(\mathbf{X})} \right|,$$

$$j = 1, 2, ..., 6 \tag{2}$$

where, Y_j (X) is the natural value of each quality indicator, j = 1, 2, ..., 6; Y_j^{pes} is the worst value of each quality indicator; $Y_{j, max}$ and $Y_{j, min}$ are the maximum and minimum values for each quality indicator.

A 2% reduction of the maximum and minimum value depending on the requirements of the relevant quality indicator $Y_j(X)$, j = 1, 2, 6 is used in the present article. The mentioned value of the quality indicators are given in Tabe 4.

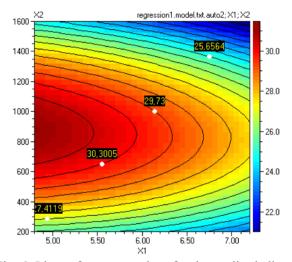


Fig. 6. Lines of constant values for the quality indicator Open porosity (Y_2) .

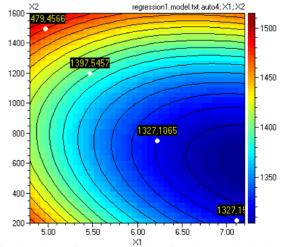


Fig. 8. Lines of constant values for the quality indicator Beginning of deformation (Y_4) .

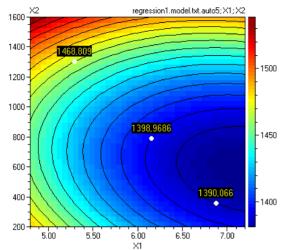
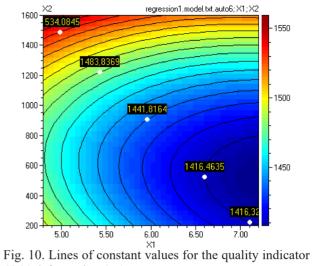


Fig. 9. Lines of constant values for the quality indicator 4 % deformation (Y₅).



40 % deformation (Y₆).

Table 4. Normalized value of the quality indicators.

No	Natural values					Normalized values						
	Y ₁	Y_2	Y_3	Y_4	Y_5	Y_6	$\Delta \overline{Y}_1 \downarrow$	$\Delta \overline{Y}_2 \downarrow$	$\Delta \overline{Y}_3 \uparrow$	$\Delta \overline{Y}_4 \uparrow$	$\Delta \overline{Y}_5 \uparrow$	$\Delta \overline{\overline{Y}}_6 \uparrow$
-	%	%	MPa	°C	°C	°C		-			-	
1	0.39	27.24	10.0	1460	1470	1490	0.882	0.319	0.054	0.750	0.600	0.571
2	0.53	28.97	12.0	1450	1480	1490	0.816	0.160	0.102	0.700	0.667	0.571
3	0.71	29.20	10.8	1440	1480	1480	0.731	0.139	0.073	0.650	0.667	0.500
4	1.20	30.60	7.7	1420	1460	1480	0.500	0.011	0.020	0.550	0.533	0.500
5	1.25	30.72	8.2	1410	1460	1480	0.476	0.020	0.012	0.500	0.533	0.500
6	1.38	30.00	8.8	1450	1480	1510	0.415	0.066	0.026	0.700	0.667	0.714
7	2.26	30.53	14.0	1460	1500	1530	0.020	0.017	0.149	0.750	0.800	0.857
8	2.25	24.89	39.7	1510	1530	1550	0.005	0.534	0.757	0.980	0.980	0.980
9	0.27	26.21	10.8	1340	1400	1420	0.939	0.413	0.073	0.150	0.133	0.071
10	0.34	27.43	12.8	1340	1400	1430	0.906	0.301	0.121	0.150	0.133	0.143
11	0.39	28.45	12.2	1330	1390	1420	0.882	0.208	0.106	0.100	0.067	0.071
12	0.90	28.71	8.6	1320	1390	1420	0.642	0.184	0.021	0.050	0.067	0.071
13	1.05	29.98	9.4	1320	1400	1440	0.571	0.068	0.040	0.050	0.133	0.214
14	1.20	29.28	11.0	1370	1420	1460	0.500	0.132	0.078	0.300	0.267	0.357
15	1.70	27.83	16.5	1420	1470	1500	0.264	0.265	0.208	0.550	0.600	0.643
16	1.99	20.00	50.0	1470	1510	1530	0.127	0.982	0.980	0.800	0.867	0.857
17	0.14	24.33	11.5	1320	1400	1420	0.980	0.585	0.090	0.050	0.133	0.071
18	0.16	24.81	13.2	1330	1400	1420	0.991	0.541	0.130	0.100	0.133	0.071
19	0.18	26.39	12.4	1320	1390	1410	0.981	0.397	0.111	0.050	0.067	0.020
20	0.74	28.03	11.7	1310	1380	1410	0.717	0.246	0.095	0.020	0.020	0.020
21	0.88	28.41	11.9	1310	1400	1430	0.651	0.212	0.099	0.020	0.133	0.143
22	1.10	27.89	12.1	1360	1410	1450	0.547	0.259	0.104	0.250	0.200	0.286
23	1.48	25.30	29.2	1390	1440	1480	0.368	0.496	0.508	0.400	0.400	0.500
24	1.45	19.80	45.4	1480	1500	1520	0.382	0.980	0.891	0.850	0.800	0.786

^{*}The arrows in the header indicate the direction of the desired values of the objective parameters

Fractional-rational generalized arithmetic mean function of usefulness, Strategy 1 (Strat-1)

Strat-1 =
$$F_{am}^{p}(\overline{y}(\mathbf{x}), \mathbf{x}) = \frac{\frac{1}{m_{1}} \sum_{j=1}^{m_{1}} \overline{y}_{j}(\mathbf{x})}{\frac{1}{m_{2}} \sum_{j=1}^{m_{2}} \overline{y}_{j}(\mathbf{x})} = \frac{\overline{y}_{3} + \overline{y}_{4} + \overline{y}_{5} + \overline{y}_{6}}{\frac{4}{\overline{y}_{1} + \overline{y}_{2}}}$$

$$= \frac{4}{\overline{y}_{1} + \overline{y}_{2}}$$
(3)

Fractional-rational generalized geometric mean objective function of usefulness, Strategy 2 (Strat-2)

$$Strat-2 \equiv F_{gm}^{p}(\overline{y}(\mathbf{x}), \mathbf{x}) = \frac{\sqrt[m_1]{\prod_{j=1}^{m_1} \overline{y}_j(\mathbf{x})}}{\sqrt[m_2]{\prod_{j=1}^{m_2} \overline{y}_j(\mathbf{x})}} = \frac{\sqrt[4]{\overline{y}_3 \cdot \overline{y}_4 \cdot \overline{y}_5 \cdot \overline{y}_6}}{\sqrt[2]{\overline{y}_1 \cdot \overline{y}_2}}$$

$$(4)$$

Fractional-rational generalized harmonic mean objective function, Strategy 3 (Strat-3)

Strat-
$$3 \equiv F_{hm}^{p}(\overline{y}(\mathbf{x}), \mathbf{x}) = \frac{m_1}{\sum_{j=1}^{m_1} \frac{1}{\overline{y}_j(\mathbf{x})}} / \frac{m_2}{\sum_{j=1}^{m_2} \frac{1}{\overline{y}_j(\mathbf{x})}} = \frac{4}{\frac{1}{\overline{y}_3} + \frac{1}{\overline{y}_4} + \frac{1}{\overline{y}_5} + \frac{1}{\overline{y}_6}} / \frac{2}{\frac{1}{\overline{y}_1} + \frac{1}{\overline{y}_2}}$$

$$(5)$$

The results for the three fractional-rational generalized functions are presented in Table 5 and presented

Table 5. Results for the fractional-rational generalized functions.

Tuncu			,		,	
No	Strat – 1	R	Strat – 2	R	Strat – 3	R
1	0.8227	12	0.6486	13	0.3692	15
2	1.0446	11	1.1220	9	1.0286	9
3	1.0858	9	1.1127	10	0.9154	10
4	1.5689	6	3.1396	3	3.3397	4
5	1.5563	7	2.0412	5	1.1517	7
6	2.1901	3	1.8444	6	0.8216	12
7	34.1728	1	28.2014	1	20.5410	2
8	3.4316	2	18.3049	2	97.5899	1
9	0.1583	18	0.1625	19	0.1668	19
10	0.2265	17	0.2608	16	0.3003	16
11	0.1580	19	0.1970	18	0.2456	17
12	0.1268	21	0.1381	21	0.1457	21
13	0.3428	15	0.4426	15	0.5788	13
14	0.7927	13	0.8462	12	0.8443	11
15	1.8919	4	1.7335	7	1.5363	5
16	1.5797	5	2.4706	4	3.8644	3
17	0.1101	22	0.1068	22	0.1037	22
18	0.1419	20	0.1441	20	0.1458	20
19	0.0899	23	0.0836	23	0.0753	23
20	0.0802	24	0.0702	24	0.0679	24
21	0.2293	16	0.2113	17	0.1680	18
22	0.5207	14	0.5214	14	0.5143	14
23	1.0461	10	1.0508	11	1.0555	8
24	1.2213	8	1.3576	8	1.5090	6

graphically on Figs. 11 - 13.

From the analysis of the information, given in Table 6 and presented on Fig. 11 to Fig. 13 it can be seen that for the fractional-rational generalized arithmetic mean and geometric mean functions, the Compromise solution No 7 has the highest rank, followed by a

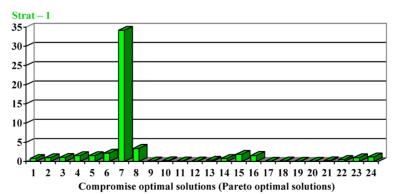


Fig. 11. Results for the fractional-rational generalized arithmetic mean function of usefulness for all quality indicators.

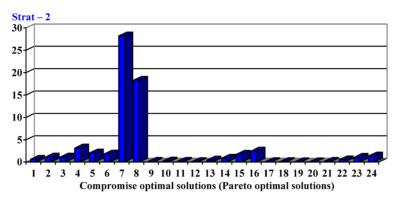


Fig. 12. Results for the fractional-rational generalized geometric mean function of usefulness for all quality indicators.

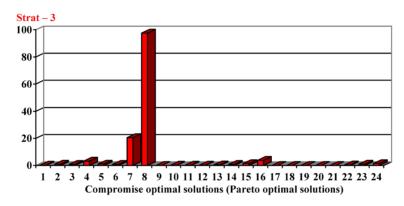


Fig. 13. Results for the fractional-rational generalized harmonic mean function of usefulness for all quality indicators.

Table 6. Values of the fractional-rational generalized functions of usefulness for strategies Strat-1, Strat-2 and Strat-3 for the Rank 1 and Rank 2 Compromise solutions.

	1					
No	Strat - 1	R	Strat - 2	R	Strat - 3	R
7	34.1728	1	28.2014	1	20.5410	2
8	3.4316	2	18.3049	2	97.5899	1
Δ	30.7412	-	9.8965	-	77.0489	-

Compromise solution No 8. For the fractional-rational generalized harmonic mean function the highest rank has a Compromise solution No 8 followed by Compromise solution No 7.

In the optimal decision making for a multi-objective optimization, the solutions with the highest two-three ranks are usually analyzed, and the analysis of the highest five-six ranks is rarely used. The proposed dominant solutions for compromise optimal decision making are No 7 and No 8, the solutions with rank 3 have significantly lower values of the ractional-rational generalized functions. The values of the fractional-rational generalized functions for the competing options

and the differences between Compromise solutions No 7 and No 8 are given in Table 6.

The difference between the two values (rank 1 and rank 2) is the largest for strategy Strat-3, and the dominant Compromise solution is No 8. For the final decision, which Compromise solution to choose will be it is necessary to analyze the economic indicators as well.

CONCLUSIONS

 The obtained experimental-statistical mathematical models describe well all six quality indicators for the "Fosuk" type driving masses with a confidence

- probability between 95 % and 99 %.
- It was found that the greatest influence on the quality indicators of the driving masses exerts the amount of Al₂O₂.
- The deformation indicators of driving masses beginning of deformation, 4 % deformation and 40 % deformation are highly correlated.
- In order to implement an optimization procedure of the six quality indicators, a transformation of the physical values of the quality indicators into dimensionless coefficients of usefulness was done.
- Three new variants of fractional-rational generalized functions of usefulness are proposed, and the results are compared.
- A ranking of 24 experimental variants for driving masses formation and a reasoned choice of two competing optimal variants were recommended.
- Three new variants of fractional-rational generalized functions of usefulness are proposed for the multicriteria optimization of driving masses and the results are compared with the classical ones based on arithmetic mean, geometric mean and harmonic mean values. The investigation of the new generalized functions show that the functions significantly enlarge potentiality for the effectiveness of the optimal decision making by giving much larger number of possible optimal decisions in dynamic changes of circumstances of prices, row materials, technologies, etc.

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