3D SPACE KINEMATICS OF A ROBOT FOR PROCESS AUTOMATION IN METALLURGY

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ABSTRACT

This paper presents the main results of a study on the kinematics in 3D space of a robot used for process automation in metallurgy.

The robot is studied as a mechanical system with twelve degrees of freedom consisting of seven rigid bodies. The transition matrices between the local and reference coordinate systems are defined in symbolic form. The position vectors and linear velocities of characteristic points are also defined in symbolic form as well as angular velocities of bodies from the mechanical system. A calculation algorithm is compiled and entered into a standard mathematical software product. Results are obtained in symbolic form and are valid for all mechanical systems with an analogous dynamic model. The kinematics results represent a basis for studying the dynamics and vibrations of a robot for automating processes in metallurgy.

Keywords: metallurgy, automation, robots, spatial kinematics.

INRODUCTION

Digitalization in modern metallurgy allows the complete automation of all installations and the use of robots in hazardous work areas, which significantly increases the safety at the workplace. The robots serve a certain area and must have such parameters that would ensure the performance of the respective process. As an example, the process of charging metallurgy furnaces with zinc-cathode plate can be given. Zinc plates are delivered in large packets near roller conveyors which are then used to move the plates to the furnaces. In this temporary warehouse they are stored in a vertical position. The robot takes from there three plates at a time and places them in a horizontal position on the roller conveyor. This process is executed fast enough for two furnaces to be charged. The robot is positioned between the temporary warehouse and the two conveyors. It must have some important characteristics to meet all requirements. These are range, speed of movement and accuracy of positioning of the operation unit, accelerations of the units of the robot as they move. These characteristics are kinematic characteristics and for a robot to be properly constructed or selected from a catalogue, the kinematics of the robot under these operating conditions has to be investigated.

When designing robots, it is necessary to use modern means and methods of geometric, kinematic and dynamic analysis and synthesis based on mechano-mathematical matrix methods [1 - 5].

From the studies and analysis of the current level of the structures and the methods for modeling and calculating robots, it can be concluded that most of the structures are built on the basis of outdated approximate methods that do not allow the optimal design. There are significant gaps in many of the works.

Oscillations have a significant impact on the dynamic loads, reliability and durability of the robot.

Existing modern methods of matrix mechanics, including finite element (FEM) methods, are useful for precise calculation of robots.

EXPERIMENTAL

Kinematic model

The kinematic model of the robot is shown in Fig. 1. It includes seven bodies. The structural scheme of the robot consists of a base 1, on which a manipulator is assembled. The manipulator consists of six bodies - 2, 3, 4, 5, 6 and 7 with the ability to perform rotations, respectively, relative to the axes O_2z_2, O_3y_3, O_4y_4, O_5x_5, O_6y_6, O_7x_7. The position and movement of the robot in space is calculated in the direction of a pre-selected starting coordinate system, which coincides with the coordinate system of body 1. Body 1 is stationary fixed to the foundation and vibrations are only calculated here. The center of the coordinate system of body 1 matches the center of mass of the body. The local coordinate systems, fixed to the bodies, have axes parallel to the axes of the reference coordinate system.

RESULTS AND DISCUSSION

Matrix methods were used to model the robot's kinematics. The results are presented in symbolic form as the calculations were performed using the standard Mathematica software [6].

The unique position of the mechanical system in space relative to the reference coordinate system $O_0x_0y_0z_0$ is determined by the vector of generalized coordinates:

$$\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6 \quad q_7 \quad q_8 \quad q_9 \quad q_{10} \quad q_{11} \quad q_{12}]^T \qquad (1)$$

The free small spatial vibrations are carried out along the generalized coordinates around an equilibrium state of the mechanical system, set with specific values of the "program" movements. Such separation of the movement is possible because the speed, with which the amplitude and direction of the vibrations change along the generalized coordinates, is significantly greater than that of the "program" movements. The generalized coordinates and their derivatives are considered small quantities and are expressed to the second order, the "program" movements are assumed to be constant values.

$$q_i = q_i^* + q_i \qquad i = 7 - 12 \qquad (2)$$

where q_i^* is program movement, q_i is small vibration.

Therefore, the generalized coordinates of the studied robot have the form:

$$q_1 = x_1;\ q_2 = y_1;\ q_3 = z_1;\ q_4 = \theta_{x1};$$

$$q_5 = \theta_{y1};\ q_6 = \theta_{z1};\ q_7 = \Phi_{32}^* + \theta_{z2};$$

$$q_8 = \Phi_{23}^* + \theta_{y3};\ q_9 = \Phi_{24}^* + \theta_{y4};$$

$$q_{10} = \Phi_{15}^* + \theta_{x5};\ q_{11} = \Phi_{26}^* + \theta_{y6};$$

$$q_{12} = \Phi_{17}^* + \theta_{x7} \qquad (3)$$

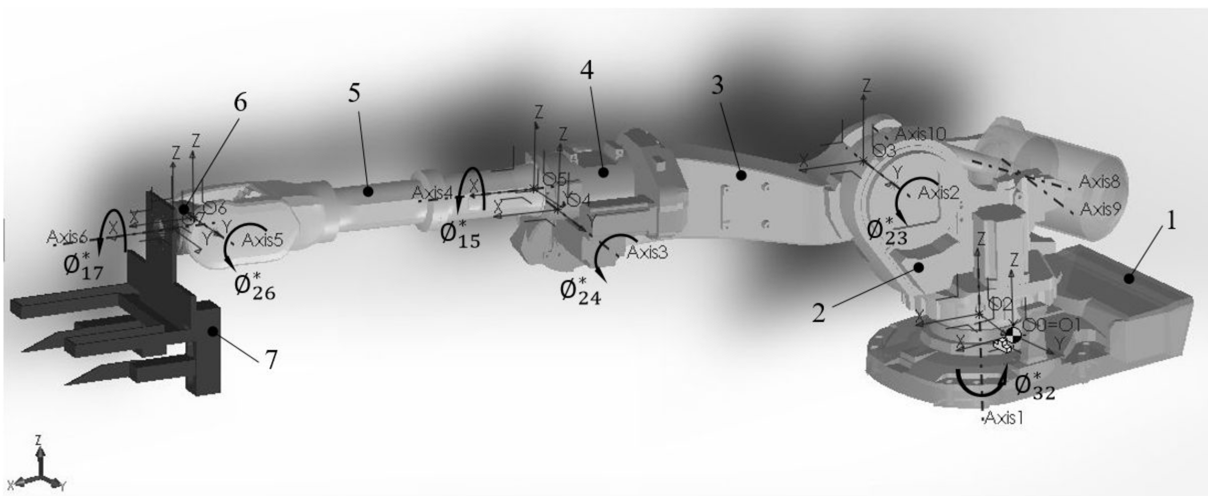

Fig. 1. The kinematic model of the robot.

Transfer Matrices

The transition from the local coordinate systems to the reference coordinate system is carried out by the transition matrices.

For body 1

$$\mathbf{A}_1^0 = \mathbf{At}_1^0.\mathbf{Au}_1^0; \tag{4}$$

where: $\mathbf{A}_1^0$ is a homogeneous transition matrix, that take into account translations and rotations; $\mathbf{At}_1^0$ is a translational transition matrix; $\mathbf{Au}_1^0$ is a rotation transition matrix;

$$\mathbf{At}_1^0 = \begin{bmatrix} 1 & 0 & 0 & l_{x01}+x_1 \\ 0 & 1 & 0 & l_{y01}+y_1 \\ 0 & 0 & 1 & l_{z01}+z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\mathbf{Au}_1^0 = \mathbf{A}_{x1}.\mathbf{A}_{y1}.\mathbf{A}_{z1};$$

$\mathbf{A}_{x1}, \mathbf{A}_{y1}, \mathbf{A}_{z1}$ are rotation transition matrices with respect to the corresponding coordinate axes.

$$\mathbf{A}_{x1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_{x1} & -\sin\theta_{x1} & 0 \\ 0 & \sin\theta_{x1} & \cos\theta_{x1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{y1} = \begin{bmatrix} \cos\theta_{y1} & 0 & \sin\theta_{y1} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_{y1} & 0 & \cos\theta_{y1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{z1} = \begin{bmatrix} \cos\theta_{z1} & -\sin\theta_{z1} & 0 & 0 \\ \sin\theta_{z1} & \cos\theta_{z1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since vibrations are small, linearization can be performed. Dependencies apply:

$\sin\theta_i = \theta_i \qquad \cos\theta_i = 1$

(Angle θ is measured in rad - a small number)

Finally for the transition matrix for body 1 is obtained:

$$\mathbf{A}_1^0 = \begin{bmatrix} 1 & -\theta_{z1} & \theta_{y1} & l_{x01}+x_1 \\ \theta_{z1} & 1 & -\theta_{x1} & l_{y01}+y_1 \\ -\theta_{y1} & \theta_{x1} & 1 & l_{z01}+z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For body 2

$$\mathbf{A}_2^0 = \mathbf{A}_1^0.\mathbf{A}_2^1 \tag{5}$$

$$\mathbf{A}_2^1 = \mathbf{At}_2^1.\mathbf{Au}_2^1;$$

$$\mathbf{At}_2^1 = \begin{bmatrix} 1 & 0 & 0 & l_{x12} \\ 0 & 1 & 0 & l_{y12} \\ 0 & 0 & 1 & l_{z12} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Au}_2^1 = \begin{bmatrix} \cos(\Phi_{32}^* + \theta_{z2}) & -\sin(\Phi_{32}^* + \theta_{z2}) & 0 & 0 \\ \sin(\Phi_{32}^* + \theta_{z2}) & \cos(\Phi_{32}^* + \theta_{z2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The results from here are with a large volume and cannot be visualized in the article. Therefore, only the calculation formulas will be given.

For body 3

$$\mathbf{A}_3^0 = \mathbf{A}_2^0.\mathbf{A}_3^2 \tag{6}$$

$$\mathbf{A}_3^2 = \mathbf{At}_3^2.\mathbf{Au}_3^2;$$

$$\mathbf{At}_3^2 = \begin{bmatrix} 1 & 0 & 0 & l_{x23} \\ 0 & 1 & 0 & l_{y23} \\ 0 & 0 & 1 & l_{z23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Au}_3^2 = \begin{bmatrix} \cos(\Phi_{23}^* + \theta_{y3}) & 0 & \sin(\Phi_{23}^* + \theta_{y3}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\Phi_{23}^* + \theta_{y3}) & 0 & \cos(\Phi_{23}^* + \theta_{y3}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For body 4

$$\mathbf{A}_4^0 = \mathbf{A}_3^0.\mathbf{A}_4^3 \tag{7}$$

$$\mathbf{A}_4^3 = \mathbf{At}_4^3.\mathbf{Au}_4^3;$$

$$\mathbf{At}_4^3 = \begin{bmatrix} 1 & 0 & 0 & l_{x34} \\ 0 & 1 & 0 & l_{y34} \\ 0 & 0 & 1 & l_{z34} \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\mathbf{Au}_4^3 = \begin{bmatrix} \cos(\Phi_{24}^* + \theta_{y4}) & 0 & \sin(\Phi_{24}^* + \theta_{y4}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\Phi_{24}^* + \theta_{y4}) & 0 & \cos(\Phi_{24}^* + \theta_{y4}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For body 5

$$\mathbf{A_5^0} = \mathbf{A_4^0}.\mathbf{A_5^4} \tag{8}$$

$$\mathbf{A_5^4} = \mathbf{At_5^4}.\mathbf{Au_5^4};$$

$$\mathbf{At_5^4} = \begin{bmatrix} 1 & 0 & 0 & l_{x45} \\ 0 & 1 & 0 & l_{y45} \\ 0 & 0 & 1 & l_{z45} \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\mathbf{Au_5^4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\Phi_{15}^* + \theta_{x5}) & -\sin(\Phi_{15}^* + \theta_{x5}) & 0 \\ 0 & \sin(\Phi_{15}^* + \theta_{x5}) & \cos(\Phi_{15}^* + \theta_{x5}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For body 6

$$\mathbf{A_6^0} = \mathbf{A_5^0}.\mathbf{A_6^5} \tag{9}$$

$$\mathbf{A_6^5} = \mathbf{At_6^5}.\mathbf{Au_6^5};$$

$$\mathbf{At_6^5} = \begin{bmatrix} 1 & 0 & 0 & l_{x56} \\ 0 & 1 & 0 & l_{y56} \\ 0 & 0 & 1 & l_{z56} \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\mathbf{Au_6^5} = \begin{bmatrix} \cos(\Phi_{26}^* + \theta_{y6}) & 0 & \sin(\Phi_{26}^* + \theta_{y6}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\Phi_{26}^* + \theta_{y6}) & 0 & \cos(\Phi_{26}^* + \theta_{y6}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For body 7

$$\mathbf{A_7^0} = \mathbf{A_4^0}.\mathbf{A_5^4} \tag{10}$$

$$\mathbf{A_5^4} = \mathbf{At_5^4}.\mathbf{Au_5^4}$$

$$\mathbf{At_5^4} = \begin{bmatrix} 1 & 0 & 0 & l_{x45} \\ 0 & 1 & 0 & l_{y45} \\ 0 & 0 & 1 & l_{z45} \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\mathbf{Au_5^4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\Phi_{15}^* + \theta_{x5}) & -\sin(\Phi_{15}^* + \theta_{x5}) & 0 \\ 0 & \sin(\Phi_{15}^* + \theta_{x5}) & \cos(\Phi_{15}^* + \theta_{x5}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position vectors

For the purpose of the research, the vectors that define the position of the mass centers of the bodies and the points of attachment of the elastic elements are needed.

$$\mathbf{R_{Ci}^0} = \mathbf{A_i^0}.\mathbf{r_{Ci}}; \tag{11}$$

$$i = 1 - 7$$

$\mathbf{R_{Ci}^0}$ is the position vector of the mass center in the reference coordinate system;

$\mathbf{r_{Ci}}$ is the vector of the mass center in the local coordinate system of the corresponding body.

The same formulas are used to define all points of the bodies.

Vectors of linear velocities of the mass centers of bodies

These vectors are needed to calculate the kinetic energy of the mechanical system in the study of dynamics.

The linear velocity vector $\mathbf{V_{Ci}^0}$ of the mass center of the respective body is obtained by differentiation with respect to time of the position vector; or as a sum of the products of the partial derivatives of the transition matrices with respect to the generalized coordinates by the corresponding generalized velocities along the entire kinematic chain, multiplied by the position vector of the center of mass in the corresponding local coordinate system.

$$\mathbf{V_{Ci}^0} = \frac{d\mathbf{R_{Ci}^0}}{dt} = \left[\sum_{k=1}^{12} \left(\frac{\partial \mathbf{A_i^0}}{\partial q_k}\right) \dot{q}_k\right].\mathbf{r_{Ci}} \tag{12}$$

$$i = 1 - 7$$

k is generalized coordinate.

Vectors of angular velocities of bodies

These vectors are also needed to calculate the kinetic energy of the mechanical system when studying dynamics. Angular velocity vectors must be projected into the local coordinate system of the corresponding body.

The vector of the absolute angular velocity of the i-th body - $\mathbf{\Omega_i^i}$, projected on the axes of the local coordinate system $O_i x_i y_i z_i$, is obtained as a sum of the projections of the absolute angular velocity of body i-1 and the relative angular velocity of body i, projected on axes of the local coordinate system.)

$$\mathbf{\Omega_i^i} = \sum_{n=1}^{i-1} \left(\mathbf{A_n^i}^{\mathbf{T}}.\mathbf{A_{\Omega n,n}}.\mathbf{\Omega_n}\right) + \mathbf{A_{\Omega i,i}}.\mathbf{\Omega_i} \tag{13}$$

where n is index of previous body.

CONCLUSIONS

When constructing or selecting a robot from a catalogue to automate metallurgy processes, the robot's 3D kinematics are investigated. This ensures the implementation of the relevant process.

The results of the kinematics study are also used to study the dynamics and oscillations of robots. All results were presented with mathematical symbols. They are valid for all robots with the same dynamic model, i.e. they can have different geometrical, mass-inertia and elastic characteristics, but the formulas and algorithm for analysis and synthesis are the same.

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