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FREE UNDAMPED SPATIAL VIBRATIONS OF A ROBOT FOR PROCESS AUTOMATION IN METALLURGY

Valentin Slavov, Verjinia Aleksandrova

University of Chemical Technology and Metallurgy
8 Kliment Ohridski Blvd., Sofia 1797, Bulgaria
E-mail: vad@uctm.edu

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ABSTRACT

This paper presents the results of mechano-mathematical modeling of free undamped spatial vibrations of a robot for process automation in metallurgy.

A dynamic model of the robot is made. It is studied as a mechanical system with twelve degrees of freedom, contained by seven rigid bodies.

Kinetic and potential energy are derived symbolically by using the kinematics study results. The matrices characterizing the mass-inertial and elastic properties of the mechanical system are obtained. The differential equations that describe the free undamped vibrations are derived. They take into account the geometrical, mass, inertial and elastic characteristics of the mechanical system.

A compiled calculation algorithm is entered into a standard mathematical software product. Results are obtained in symbolic and graphical form. The natural frequencies and natural mode of a robot with concrete parameters are determined.

Results of the study of free undamped vibrations represent a basis for studying of the free damped and forced vibrations of a robot for process automation in metallurgy.

Keywords: automation, metallurgy, robots, dynamics, Free Spatial Vibrations.

INTRODUCTION

In metallurgy, production processes are continuous. Charging metallurgy furnaces with zinc-cathode plates is also a continuous process. This process takes place in a hazardous environment for human health – high temperature, moving plates with a large mass of up to 100 kg. Automation is carried out with a robot that must be constructed or selected from a catalogue, so that to fulfill all the requirements for work in such an environment and to carry out the implementation of the process reliably and durably. Oscillations are known to have a substantial impact on dynamic loads, reliability, and durability. When the robot works in resonance, even destruction of the structure is possible.

When designing robots, it is necessary to study the free vibrations in order to determine the coefficients of elasticity in the connections between individual bodies. They must be set so that the natural frequencies of the robot bodies are different from the frequencies of the forced vibrations to avoid the phenomenon of resonance.

For this purpose, modern tools and methods for dynamic analysis and synthesis based on mechano-mathematical matrix methods are used [1 - 4].

The research includes development of an algorithm for deriving the differential equations that describe the free undamped vibrations of the mechanical system and determining the natural frequencies and natural mode. This is done with Mathematica software [5].

EXPERIMENTAL

Dynamic model

The dynamic model of the robot is shown in Fig. 1. The mechanical system consists of seven rigid bodies interconnected by respective links and elastic elements. All bodies are characterized by their masses m_i and tensors of mass moments of inertia $\mathbf{J}_i$, i = 1-7.

$$\mathbf{J_i} = \begin{bmatrix} J_{ixx} & -J_{ixy} & -J_{ixz} & 0 \\ -J_{ixy} & J_{iyy} & -J_{iyz} & 0 \\ -J_{ixz} & -J_{iyz} & J_{izz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Body 1 is attached to the foundation by four elastic elements that have linearized dynamic coefficients of c_{xk01}, c_{yk01}, c_{zk01}, k = 1-4.

Body 3 is driven by a hydraulic cylinder, taking into account the coefficient of elasticity of the fluid. The remaining bodies are driven by electric motors, considering the coefficients of angular elasticity (N m) rad^{-1}.

The mechanical system has twelve degrees of freedom. The coordinate systems used in the dynamic model are identical to those indicated in the kinematic model [6].

Kinetic energy

Kinetic energy is calculated as the sum of the kinetic energies of all bodies.

$$E_K = \sum_{i=1}^{7} E_{Ki}(q_i, \dot{q}_i) =$$

$$\sum_{i=1}^{7} \frac{1}{2} . \left(m_i . \mathbf{V_{Ci}^0}^T . \mathbf{V_{Ci}^0} + \mathbf{\Omega_i^i}^T . \mathbf{J_i} . \mathbf{\Omega_i^i} \right) \quad (2)$$

V_{Ci}^{0} is the velocity vector of the mass center of the corresponding body, projected in the reference coordinate system; Ω_i^i is the angular velocity vector of the corresponding body, projected into the body-related (local) coordinate system.

Potential energy

Potential energy E_P is calculated as the sum of:
- the potential energy from the deformation of all elastic elements E_{PD};
- the potential energy from the force of the weight of each body E_{PG}.

$$E_P = E_{PD} + E_{PG} \quad (3)$$

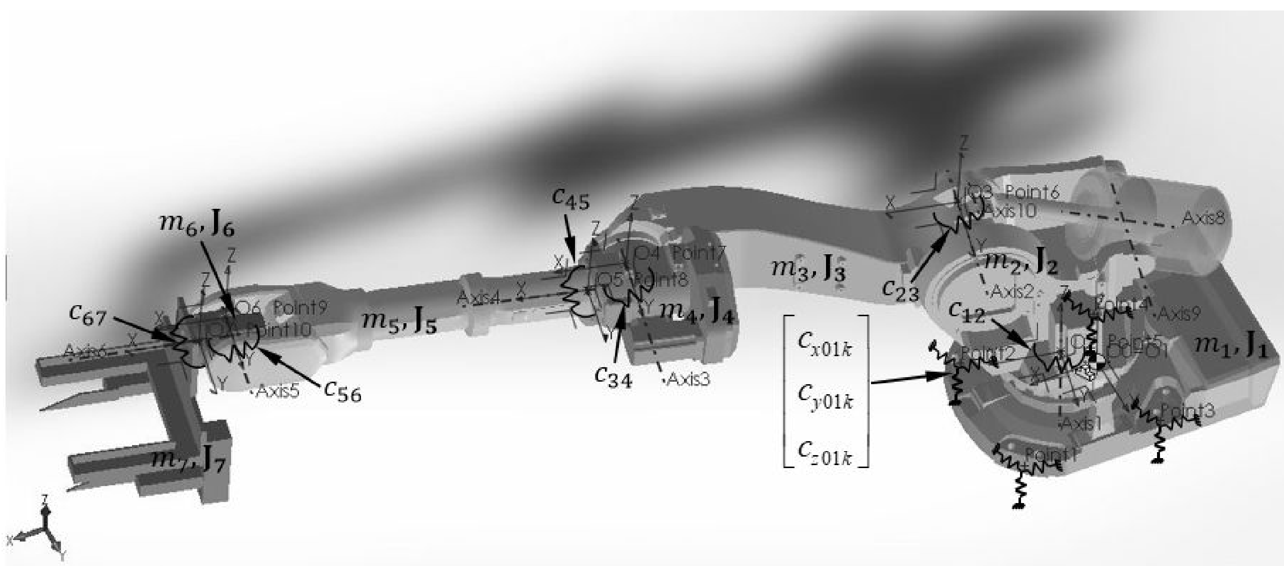

Fig. 1. Dynamic model.

$$E_{PD} = \sum_{k=1}^{10} \frac{1}{2} \left(\mathbf{c_k}^T . \mathbf{\Delta R_k}^2 \right) \tag{4}$$

c_k is the elasticity coefficient of the corresponding elastic element; ΔR_k is the deformation of the corresponding elastic element.

$$E_{PG} = \sum_{i=1}^{7} -m_i . \left(\mathbf{g}^T . \mathbf{R}_{Ci}^0 \right) \tag{5}$$

$\mathbf{g} = [0 \ \ 0 \ \ g \ \ 0]^T$

Vector **g** defines the gravitational acceleration projected into the reference coordinate system; $\mathbf{R}_{Ci}^0$ is the position vector of the mass center of the corresponding body projected in the reference coordinate system.

Differential equations of free undamped vibrations

The differential equations, which describe the free vibrations, are deduced using the Lagrange's method. This method provides best opportunities.

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}}\right) - \frac{\partial E_K}{\partial q} + \frac{\partial E_P}{\partial q} = 0 \tag{6}$$

where E_K and E_P are respectively the kinetic and the potential energy of the systems.

The obtained system of linear differential equations, which describes the free vibrations of the mechanical system is:

$$\boldsymbol{M}.\ddot{\boldsymbol{q}} + \boldsymbol{C}.\boldsymbol{q} = 0 \tag{7}$$

The matrix, which characterizes the mass-inertial properties M and the elastic properties C of the mechanical system, are:

$$\boldsymbol{M} = \left[a_{ij}\right]_{12\times 12} ; \ a_{ij} = \frac{\partial^2 E_K}{\partial \dot{q}_i . \partial \dot{q}_j} \tag{8}$$

$$\mathbf{C} = \left[c_{ij}\right]_{12\times 12} ; \ c_{ij} = \frac{\partial^2 E_P}{\partial q_i . \partial q_j} \tag{9}$$

Particular solutions to the system of the differential equation (7) are searched as,

$$q_r = h_r . \sin(\omega_r . t + \varphi) \tag{10}$$

where h_r is the amplitude of the small vibration on the generalized coordinate q_r with natural frequency ω_r, and φ is the initial phase.

After differentiation of (10) and substituting in (7), a system of linear algebraic equations is obtained. In the matrix description they are:

$$|\mathbf{C} - \omega^2 . \mathbf{M}| . \mathbf{V} = 0 \tag{11}$$

To determine the natural frequencies and the natural mode, it is necessary to solve the task about finding the natural values and the natural vectors of the equations (11). The satisfaction of the equations (11) requires the following:

$$\det|\mathbf{C} - \omega^2 . \mathbf{M}| = 0 \tag{12}$$

The roots of the characteristic's equation determine the natural frequencies. The natural frequencies form the matrix of the natural values are:

$$\omega = \mathrm{diag}\left[\omega_{r,r}\right] \qquad r = 1 \div 12 \tag{13}$$

and in [Hz]

$$f_r = \frac{\omega_{r,r}}{2\pi} \ \mathrm{Hz} \tag{14}$$

The natural values of the system (11) determine the natural vectors of the mechanical system.

Each natural frequency w_r corresponds to a vector of natural forms $\mathbf{v}_r$, which sets the ratio between the amplitudes of the vibrations. The components of the vectors define the matrix of mode vectors (modal matrix) of the system (11), which has the form:

$$\mathbf{V} = \left[\mathbf{v}_{r,j}\right]_{12\times 12} \tag{15}$$

where

$$\mathbf{v_r} = [v_{r,1} \quad v_{r,2} \quad v_{r,3} \quad v_{r,4} \quad v_{r,5} \quad v_{r,6} \quad v_{r,7} \quad v_{r,8} \quad v_{r,9} \quad v_{r,10} \quad v_{r,11} \quad v_{r,12}]$$

is the natural mode vector on the generalized coordinate for r[th] natural frequency.

RESULTS AND DISCUSSIONS

For specific values of parameters of the mechanical system, presented in the Table 1 and determined by a CAD program Solid Works, numerical results for the natural frequencies and natural mode are obtained. They are visualized with a 3D graphics Fig. 2, from which the relative values of the amplitudes at a corresponding natural frequency and for a corresponding coordinate are reported.

Table 1. Parameters of the mechanical system.

Body №	Mass, kg	Mass inertia moments, kg m^2						Mass center's coordinates, m		
	m	J_{xx}	J_{yy}	J_{zz}	J_{xy}	J_{yz}	J_{xz}	l_{Cx}	l_{Cy}	l_{Cz}
1	457.72	28.57	38.48	63.58	0.44	0.32	1.78	0	0	0
2	612.16	47.26	47.99	52.47	4.14	0.03	9.87	0.123	-0.058	0.324
3	201.17	5.09	41.08	40.82	6.06	0.41	1.71	0.761	0.047	0.007
4	303.29	11.46	11.11	13.23	2.84	2.71	1.59	-0.086	0.270	0.142
5	108.84	1.55	19.5	20.05	0.51	0	0.01	0.825	0	0
6	36.4	0.19	0.31	0.34	0.01	0	0	0.032	0.098	0
7	80.19	5.69	3.71	5.05	0	0.01	0.2	0.091	0	-0.302

Coordinates of suspension points of elastic elements			
In the coordinate system of body 1			
Point	l_{xi}, m	l_{yi}, m	l_{zi}, m
1	0.425	0.271	-0.100
2	0.425	-0.284	-0.100
3	-0.126	-0.283	-0.100
4	-0.126	-0.296	-0.100

Elasticity coefficients			
Between bodies	c_{xi},N m^{-1}	c_{yi}, N m^{-1}	c_{zi}, N m^{-1}
0 and 1	1500000	1500000	2000000

Elasticity coefficients. (N m) rad^{-1}	
1 and 2	C_{120z}= 1000000
2 and 3	C_{230y} = 1000000
3 and 4	C_{340y} = 1000000
4 and 5	C_{450x} = 1000000
5 and 6	C_{560y} = 1000000
6 and 7	C_{670x} = 1000000

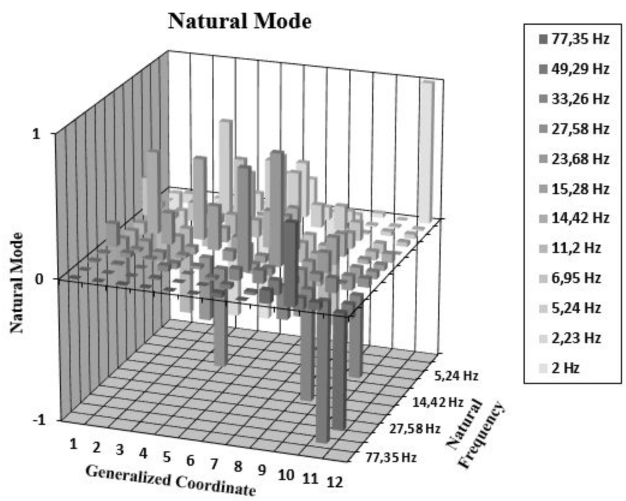

Fig. 2. Natural mode.

CONCLUSIONS

An original dynamic model is created to study the free spatial vibrations of a robot for automating processes in metallurgy. An algorithm is developed for deriving the differential equations that describe the free undamped vibrations of the mechanical system. The natural frequencies and natural mode are determined. By determining the natural frequencies, the resonance zones that must be avoided during robot operation are determined. The natural mode show at which coordinates the maximum amplitudes are obtained for the corresponding natural frequencies.

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