

## ELASTOHYDRODYNAMIC EFFECTS IN LUBRICATION OF JOURNAL BEARINGS UNDER TURBULENT CONDITIONS

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### ABSTRACT

*The current paper is aimed to investigate the deformability effects of the bearing surface on the lubrication performance of cylindrical journal bearing with finite length under turbulent conditions. The bush is coated with a thin resilient layer with a smooth surface, the radial distortions of which are of the same order of magnitude as the film thickness. The elastohydrodynamic problem is studied under elasticity conditions in accordance with the plain strain hypothesis (Winkler model). The modified Reynolds equation is carried out on the base of the turbulent lubrication model of Hirs (bulk flow model). The partial differential equation is solved numerically by successive over-relaxation technique on a finite difference grid, considering different values of coating materials, lubricant viscosity, and journal speed. Results for the performance characteristics of the bearing are given for several selected values of Reynolds number and elasticity parameters of the soft liner on the bush.*

*Keywords:* elastohydrodynamic lubrication, turbulence, journal bearing.

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### INTRODUCTION

It is well known that journal bearings are used widely and successfully for thousands of years and probably they are one of the most used machine elements in our civilization [1, 2]. Nowadays, large numbers and various types of sliding bearings have found an application in industrial machinery and equipment, transportation industry, chemical and metallurgy industry and others [2 - 5].

Journal bearings lubricated with low viscosity oils and running at high speed operate at large Reynolds numbers. In this case, the flow of a thin fluid film in the bearing becomes turbulent, so the turbulent phenomenon must be taken into account in the study of the performance of the sliding bearing.

Several researchers developed turbulent lubrication theories for hydrodynamic (HD) journal bearings. There are many papers dealing with the lubrication of turbulent

bearings [6 - 13] but a small number of them [14 - 20] discuss in detail the effect of the turbulent lubricating film on the performance characteristics of finite length journal bearings.

Along with that, it is known that for better performance of the journal bearing sometimes its surfaces are lined with materials that are much softer than the usual bearing metals. In some applications under high loading conditions of the bearings and/or when using bearings with layers on the contact surfaces, the elastic deformations affect the clearance space geometry, and this effect is significant and cannot be neglected [21 - 30].

With reference to all mentioned, the aim of the current paper is to study the simultaneous effect of the elastic deformations of the bush soft liner and turbulent conditions of the fluid film flow on the performance of elastohydrodynamic (EHD) journal bearings.

The problem considered here is studied for a Newtonian incompressible lubricant under isothermal and isoviscous conditions.

The geometrical configuration of the journal bearing with finite length is shown in Figure 1. It is assumed that both contact surfaces are smooth and have a strong circular form, the journal rotates with a constant angular velocity about its axis, and the constant external load is applied in a vertical direction. The bush of bearing is covered with a thin resilient layer, as its radial displacements are of the same order of magnitude as the film thickness. These elastic deformations are determined in accordance with the plain strain hypothesis [2, 23, 26 - 28].

The effects of turbulence are considered by the modified Reynolds equation governing the film pressure, as the latter is derived on the base of the bulk flow model of Hirs. This turbulent model is used here following the recommendations, presented in [3], where is noticed that the bulk flow theory of Hirs is to be preferred over Constantinescu's model.

The numerical solution with finite difference method is carried out to simultaneously solve the Reynolds, elasticity and film geometry equations, respectively for pressure distribution, deformations and film thickness within the lubricated clearance. Based on the EHD solution, bearing performance characteristics as load carrying capacity, Sommerfeld number, attitude angle, friction force and friction coefficient are calculated.

## MATHEMATICAL MODELING

### Modified Reynolds equation

The partial differential equation that governs hydrodynamic lubrication for two-dimensional incompressible thin fluid films with squeeze action is obtained from the integral form of continuity equation at assumption for high values of the 'global' Reynolds number  $Re = (\omega r c / \nu) > 2 \cdot 10^3$  [3, 14, 15, 17]. In the case of dynamically loaded plane journal bearing under turbulent conditions the modified Reynolds equation can be presented as [3]:

$$\frac{\partial}{\partial x} \left( G_x \frac{h^3}{\eta} \frac{\partial \bar{p}}{\partial x} \right) + \frac{\partial}{\partial z} \left( G_z \frac{h^3}{\eta} \frac{\partial \bar{p}}{\partial z} \right) = 6\omega r \frac{\partial h}{\partial x} + 12 \frac{\partial h}{\partial t} \quad (1)$$

Here  $h$  is the fluid film thickness,  $\bar{p}$  - mean turbulent

hydrodynamic pressure,  $r$  - journal radius,  $t$  - time,  $x, z$  - orthogonal Cartesian coordinates,  $\omega$  - angular velocity,  $\eta$  - fluid dynamic viscosity. The introduced in Eq. (1) turbulence coefficients  $G_x$  and  $G_z$  are function of the local Reynolds number  $Re_h = \omega r h / \nu$ . By the bulk flow model of Hirs, they are represented as:

$$G_x = \frac{1}{2 + m_0} G_z; \quad (2)$$

$$G_z = \frac{2^{1+m_0}}{n_0} Re_h^{-(1+m_0)}, \quad (3)$$

where  $m_0, n_0$  are coefficients of pressure flow. In [3, 8] are recommended the values  $m_0 = -0,25$ ,  $n_0 = 0,066$  for flow with  $Re \leq 10^5$  and smooth bearing surfaces, as in this way Eq. (2) and Eq. (3) transform to the following approximate formulas:

$$k_x \equiv \frac{1}{G_x} = 0,0687 Re_h^{0,75}; \quad (4)$$

$$k_z \equiv \frac{1}{G_z} = 0,0392 Re_h^{0,75}. \quad (5)$$

Therefore, Eq. (1) can be rewritten in a form

$$\frac{\partial}{\partial x} \left( \frac{h^3}{k_x \eta} \frac{\partial \bar{p}}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{k_z \eta} \frac{\partial \bar{p}}{\partial z} \right) = 6\omega r \frac{\partial h}{\partial x} + 12 \frac{\partial h}{\partial t}, \quad (6)$$

which is equivalent to the modified Reynolds equation developed in [3, 13], where the turbulence coefficients  $k_x$  and  $k_z$  are introduced according to the model of Constantinescu.

For laminar flow  $G_x = G_z = 0$  and from Eq. (1) the classical Reynolds equation can be obtained.

For the numerical solution, the Eq. (1) must be represented in dimensionless form as follows:

$$\begin{aligned} \bar{G}_x \frac{\partial^2 \Pi}{\partial \theta^2} + \bar{G}_x \frac{3}{H} \frac{\partial H}{\partial \theta} \frac{\partial \Pi}{\partial \theta} + \alpha \bar{G}_z \frac{\partial^2 \Pi}{\partial z_1^2} + \\ + \alpha \bar{G}_z \frac{3}{H} \frac{\partial H}{\partial z_1} \frac{\partial H}{\partial z_1} = \frac{1}{H^3} \frac{\partial H}{\partial \theta} + \frac{1}{H^3} \frac{\partial H}{\partial \tau}. \end{aligned} \quad (7)$$

In the non-dimensional form of the modified Reynolds equation (7), coordinate transformations and corresponding non-dimensional variables are introduced according to the following substitutions:  $\alpha = 2r / L$  -

diameter to length ratio,  $\beta = c / r$  - clearance ratio ( $c$  - radial clearance),  $\varepsilon = e / c$  - eccentricity ratio ( $e$  - eccentricity);  $\Pi = p(c^2 / 6\eta U r)$  - dimensionless HD pressure;  $H = h / c$  - non-dimensional film thickness;  $L$  - bearing axial length;  $U = \omega r$  - journal circumferential velocity;  $\tau = t \omega / 2$  - non-dimensional time. The coordinate transformations are related to  $\theta = x / R \approx x / r$  (bearing circumferential coordinate) and  $z_1 = 2z / L$  (dimensionless axial coordinate).

### Film thickness and elasticity equations

In the present study, it is assumed that the deformations of the layer on the bush caused by the generated hydrodynamic pressure should be superimposed on the oil film thickness. In the case considered, the film thickness equation is modified to account for the estimated elastic deformations as follows:

$$h(x, z, t) = c + e \cos \theta + \delta(x, z, t). \quad (8)$$

The last term of this equation takes into account the influence of the elastic layer deformations.

Since the lining on the bush is thin compared to the dimensions of the bearing, it is possible to determine the radial displacements of points on the liner's surface in accordance with the the plain strain hypothesis, also called Winkler model [2, 23, 26, 28]:

$$\delta = \frac{(1 + \mu)(1 - 2\mu)}{(1 - \mu)} \frac{d}{E} p, \quad (9)$$

where  $\mu$  is Poisson's ratio,  $E$  - Young's modulus,  $d$  - bearing liner thickness.

For the numerical solution, the film thickness equation (8) and the elasticity equation (9) are modified to the following dimensionless forms, respectively:

$$H = 1 + \varepsilon \cos \theta + \bar{\delta}, \quad (10)$$

$$\bar{\delta} = \frac{6\eta \omega r^2}{c^2} \frac{(1 + \mu)(1 - 2\mu)}{(1 - \mu)} \frac{d}{E} \Pi. \quad (11)$$

### Bearing performances

The resultant load carrying capacity of the bearing can be calculated by integrating the film pressure over the film region

$$\bar{W} = \sqrt{\bar{W}_1^2 + \bar{W}_2^2} = \frac{\beta^2}{6\eta \omega r L} W, \quad (12)$$

where  $\bar{W}_1$  and  $\bar{W}_2$  are radial and tangential components of the hydrodynamic film forces acting on the system and they are given respectively as:

$$\bar{W}_1 = - \int_{-1}^1 \int_0^{2\pi} \Pi \cos \theta d\theta dz_1; \quad (13.a)$$

$$\bar{W}_2 = \int_{-1}^1 \int_0^{2\pi} \Pi \sin \theta d\theta dz_1. \quad (13.b)$$

The Sommerfeld number that represents in fact the load carrying capacity coefficient in the current analysis is expressed quantitative as:

$$S = \frac{W \beta^2}{\eta \omega r L}. \quad (14)$$

As a result, also the attitude angle can be evaluated by

$$\gamma = \tan^{-1} (\bar{W}_2 / \bar{W}_1). \quad (15)$$

On the other side, by integrating the shear stress  $\tau_{xy}$  around the journal surface, the dimensionless friction force acting on the journal can be found

$$\bar{F} = \int_0^1 \int_0^{2\pi} \left. \frac{\partial u}{\partial y_1} \right|_{y_1=H} d\theta dz \quad (16)$$

and the friction coefficient can subsequently be calculated and written as

$$C_F = \frac{\bar{F}}{\bar{W}}. \quad (17)$$

### NUMERICAL SOLUTION

The elastohydrodynamic problem thus formulated involves solving the modified Reynolds equation for turbulent flow (7), the film thickness equation (10), and the elasticity equation (11) simultaneously.

At the beginning, the fluid film thickness is calculated without considering the deformations of the coating on the bearing bush. Subsequently, with this film thickness, the modified Reynolds equation (7) is numerically solved, and the pressure generated in the lubricating film

is determined. With this pressure, the resulting elastic deformations of the coating are calculated by means of equation (11). The fluid film geometry equation (10) is then recalculated incorporating the deformations already determined. The iterative procedure is repeated until the pressure convergence is achieved.

In the present work, the numerical solution of the dimensionless modified Reynolds equation is carried out using the finite difference method. In order to improve the convergence rate, the Gauss-Seidel method with an successive over-relaxation procedure is also applied.

In the current analysis, the Reynolds boundary conditions for the hydrodynamic pressure in the fluid film are used, namely: the pressure is equal to zero in those zones of the journal bearing where the gradient of pressure in the circumferential direction becomes zero, as well as the pressure is equal to zero at the axial edges of the bearing.

Based on the pressure solution found, the performance parameters such as load carrying capacity, Sommerfeld number, attitude angle, friction force and friction coefficient can be calculated.

## RESULTS AND DISCUSSION

The numerical results of the performance characteristics of EHD journal bearings in turbulent flow obtained from the theories previously mentioned are discussed in the following. In the calculations, the following basic operating parameters were used, which correspond to the presented mathematical model and, accordingly, allow analysis of the studied effects: for laminar flow –  $Re = 1 \times 10^3$ ; for turbulent flow -  $Re = 4 \times 10^3$ ,  $Re = 9 \times 10^3$  and the elasticity parameters:  $E = 2.1 \times 10^{11}$  [Pa],  $\mu = 0.25$  (steel - rigid case);  $E = 4.7 \times 10^7$  [Pa],  $\mu = 0.41$  (elastomer - soft case). The results are obtained for diameter to length ratio  $\alpha$  equal to 1.0 and 1.5 whereas the eccentricity ratio  $\varepsilon$  was varied from 0.1 to 0.9.

The effect of turbulent flow on the maximum values of the HD pressure for the “soft” case, when the surfaces deformability is taken into consideration, is shown in Fig. 2. It can be seen that a bearing operating at higher Reynolds number also has a higher maximum HD pressure. At larger eccentricities, the change in fluid pressure becomes appreciable, especially for the turbulent regime. It is evident that the film pressure

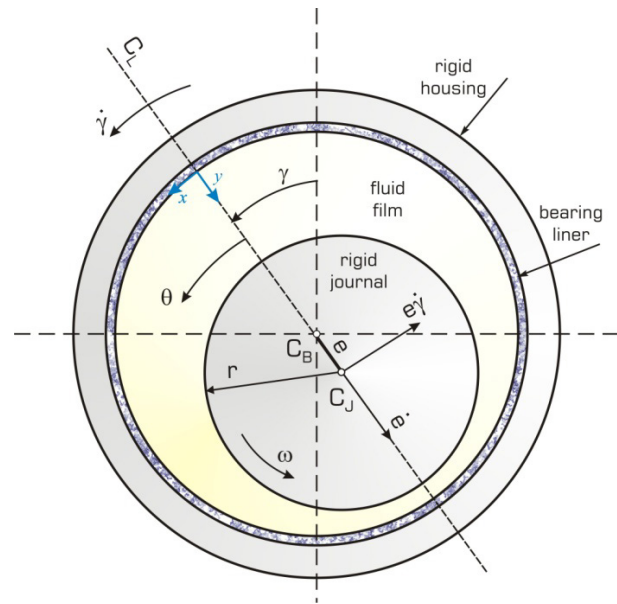


Fig. 1. Journal bearing configuration.

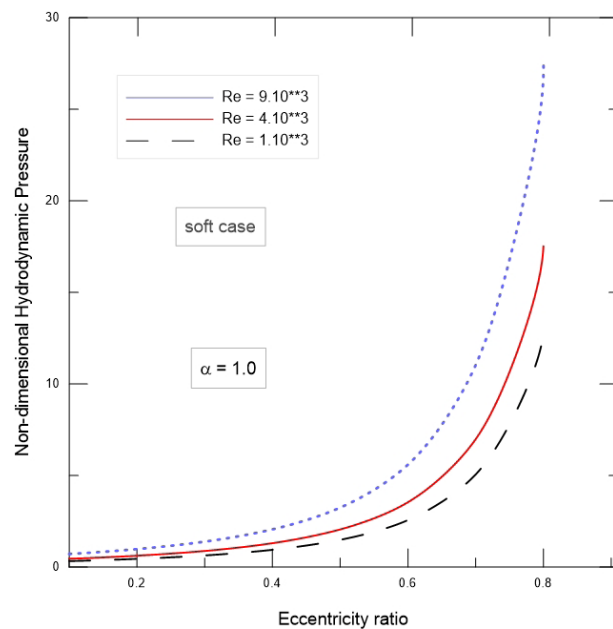


Fig. 2. Maximum pressure versus eccentricity ratio under laminar and turbulent conditions.

increases when passing from laminar to turbulent flows, such as this increase becomes more pronounced at larger values of the mean (global) Reynolds number. This is due to an increase in the apparent viscosity, which is caused by a decrease in the turbulence coefficients, accompanied by an increase in the Reynolds number.

The hydrodynamic pressure distribution under turbulent and laminar conditions (different Reynolds

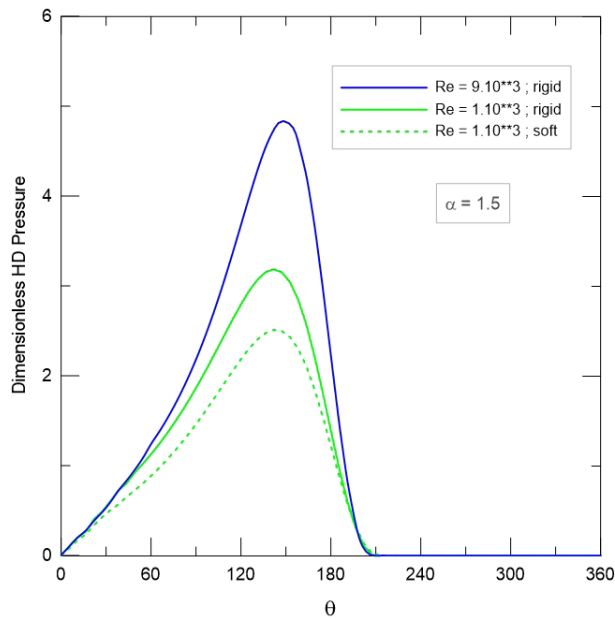


Fig. 3. Pressure distribution under different  $Re$  numbers and bush layer deformability.

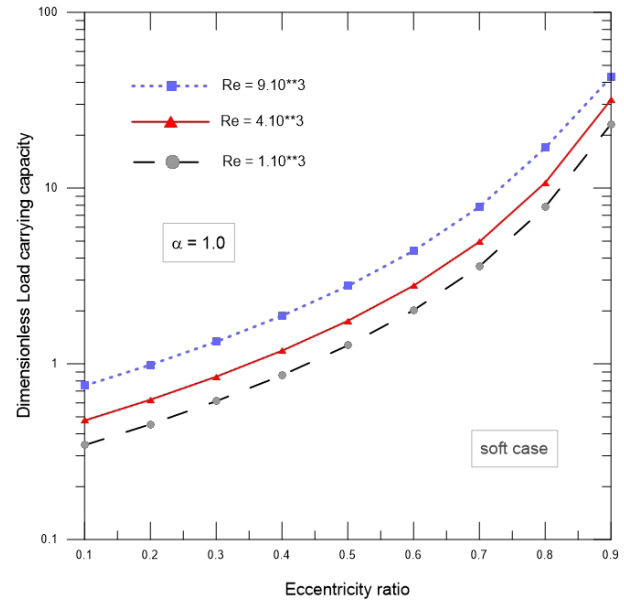


Fig. 4. Load - carrying capacity versus eccentricity ratio.

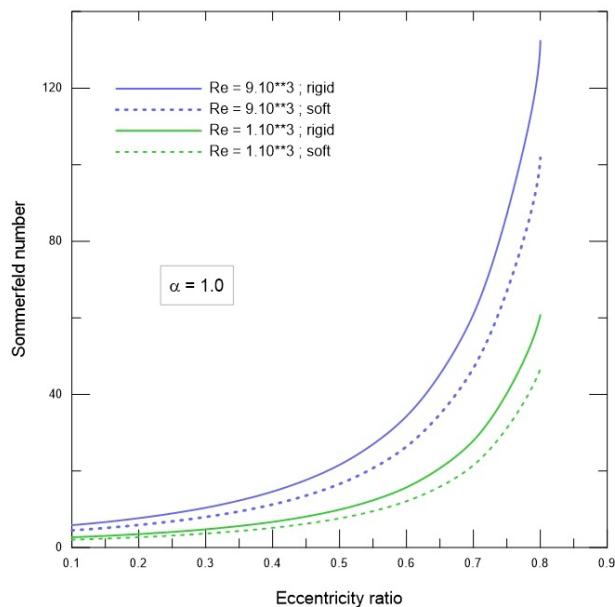


Fig. 5. Sommerfeld number versus eccentricity ratio under laminar and turbulent conditions.

numbers) for the so called in the paper “soft” and “rigid” cases is shown in Fig. 3. Analogously to the results in Fig. 2, the HD pressure values are larger for the turbulent flow compared to the laminar one, but the pressure values decrease when the deformability of the surfaces is taken into account (soft case).

The numerical results in Fig. 4 provide the variation of the dimensionless load carrying capacity with

different eccentricity ratios for the soft (elastic) case in laminar and turbulent regimes. It has been observed that as the Reynolds number increases as well as the eccentricity ratio, the load carrying capacity increases apparently.

The dependence of Sommerfeld number on the eccentricity ratio for the rigid and soft (elastic) cases under laminar and turbulent conditions is given in Fig. 5. It is found that the values of load carrying capacity coefficient increase with increasing Reynolds number for both cases, considered here, in terms of deformability. Furthermore, the Sommerfeld number increase with increasing the eccentricity ratio, as this effect is more pronounced for the middle and especially for the high eccentricity ratios. On the other hand, it is evident that the values of the load carrying coefficient decrease visibly for the soft cases where the deformations of the coating on the bearing bush are considered.

Since the Sommerfeld number is directly related to the load carrying capacity and HD pressure, it can be concluded that the maximum pressure in the bearing increases with the Reynolds number as opposed to the greater deformability of the contact surfaces. When the Reynolds number is within the laminar conditions, the pressure distribution in the bearing represents the case of „pure“ elastohydrodynamic lubrication, and if the elastic deformations are ignored, the lubrication is „purely“ hydrodynamic, for which the classical Reynolds equation is valid.



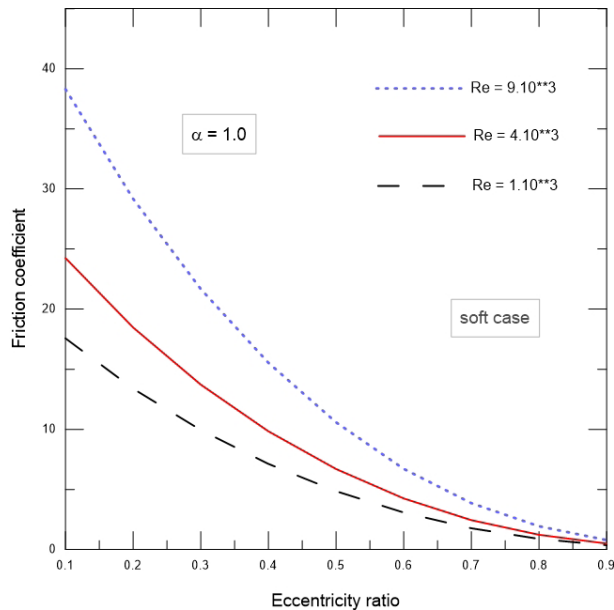


Fig. 6. Friction coefficient versus eccentricity ratio under laminar and turbulent conditions.

Fig. 6 shows results for the coefficient of friction as a function of eccentricity ratio for laminar and turbulent flow concerning the soft case in terms of deformability. As can be seen in this figure, the turbulence of the fluid flow leads to increased values of the friction coefficient, so the visibility of the turbulent effects is significant, especially at lower eccentricities.

The same effect of turbulent flow on the values of friction coefficient is presented in Fig. 7 as in addition the effect of deformability of the bush layer is taken into account. It is obvious that if the elastic deformations are not taken into account (rigid case), the friction coefficient increases compared to the soft (elastic) case, and it is normal to expect the same trend for both values of the Reynolds number shown (as well as for the entire range of possible values of the eccentricity ratio, also of the diameter to length ratio). The difference between the rigid and elastic cases gradually increases with the eccentricity ratio.

## CONCLUSIONS

In the presented analysis, the performance of HD plain bearings is investigated considering the turbulent conditions and the deformability of the elastic layer of the bearing bush. To account for turbulent effects, the

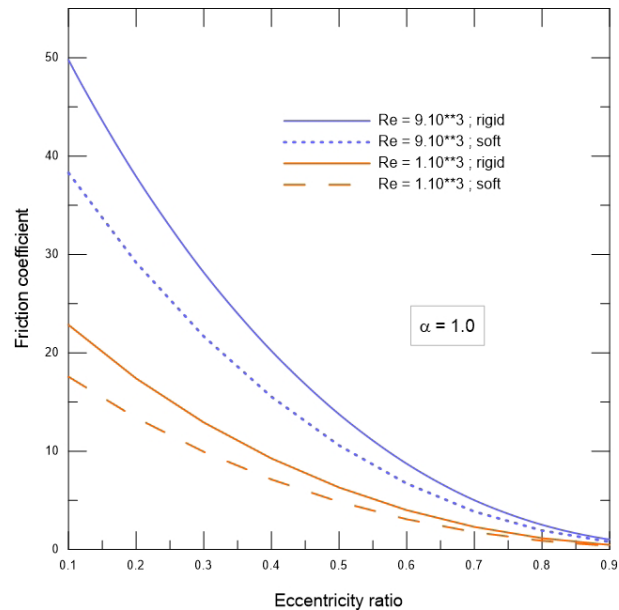


Fig. 7. Effect of layer deformability on the friction coefficient.

modified Reynolds equation is solved simultaneously with the film thickness and elasticity equations using FDM. Based on the values of HD pressure and film thickness, performance characteristics such as load capacity, attitude angle, friction force and friction coefficient are calculated.

According to the results obtained in the present research, the conclusions are as follows.

The calculated results based on the turbulent lubrication equation are in good agreement with those published previously. Since the film pressure in turbulent flows is greater than in laminar flows, it follows that the load carrying capacity and Sommerfeld number are greater for the same eccentricity ratios. This trend is more pronounced with increasing Reynolds number. On the other hand, pressure and load carrying capacity increase when the deformability of the contact surfaces (of the bush layer) decreases. The frictional force and the friction coefficient increase in turbulent flow. If the fluid film flow in the bearing is laminar, the problem reduces to the classical EHD lubrication case. Both of the effects studied here, turbulent conditions and elastic deformations of the bearing surfaces must be taken into account when solving such problems, since this change in HD pressure values affects the static and dynamic performance characteristics of the journal bearings.

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