EFFECT OF THE LIMITING DEFORMATION ZONE UNDER CONDITIONS OF ASYMMETRIC LOADING DURING ROLLING OF MEDIUM THICKNESS STRIPS

Valery Chigirinsky[1], Abdrakhman Naizabekov[1], Sergey Lezhnev[1], Sergey Kuzmin[1], Evgeniy Panin[2], Andrey Tolkushkin[1], Irina Volokitina[2], Rozina Yordanova[3], Marcin Knapinski[4]

[1]Rudny Industrial Institute, Rudny, Kazakhstan
[2]Karaganda Industrial University, Temirtau, Kazakhstan
[3]University of Chemical Technology and Metallurgy, Sofia 1797, Bulgaria
[4]Częstochowa University of Technology, Częstochowa, Poland
Email: sergey_legnev@mail.ru

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ABSTRACT

The aim of the work is to develop a physical and mathematical model of the process under complex asymmetric loading in conditions of single-area and two-area deformation zone during plastic processing of medium-thickness strips. The stress state in case of loss of stability during rolling of strips of medium thickness was investigated. The patterns of changes in the stress state of the strip under conditions of reach of the limiting deformation zone, as well as the effects of plastic shaping determined by a decrease in contact stresses under conditions of increasing deformation loading, are revealed. The described method is a visual approach to assessing the stress state of a plastic medium under conditions of complex interaction and asymmetric loading.

Keywords: mathematical model, rolling, asymmetric loading, limiting deformation zone, stress state.

INTRODUCTION

The limiting deformation zone during rolling corresponds to the deformation regime, which determines the loss of stability of the process, i.e. the slip of the strip in the rolls. Studies show that the maximum compressions, which are characterized by the maximum gripping ability of the rolls, correspond to normal stresses, which have a noticeable decrease in the lag zone or, during slipping, along the entire length of the deformation zone [1, 2]. Under conditions of a limiting deformation zone, it was found that the trapping ability of rolls can be accompanied by a single-area deformation zone, with zero or negative advance [3 - 5]. Such reserve of the rolls capture ability requires justification and explanation.

It is of theoretical and practical interest to study the stress-strain state of a metal in processes with a near-limit deformation zone. The study of these features makes it possible to identify stable modes of rolling strips of medium thickness in conditions of maximum productivity of a rolling mill with minimal force loading. The optimal solution to this problem is to create analytical solutions based on modern approaches in the theory of plasticity. To do this, it is necessary to determine the physical and mathematical model of asymmetric loading under conditions of a limiting deformation zone, to investigate the process of strip slipping in rolls using methods for solving problems of continuum mechanics [6 - 8].

The purpose of the work is to develop a physical and mathematical model of the process under conditions of the limiting deformation zone, to identify the effects of plastic shaping in the reach areas of the limiting deformation zone, using the method of functions argument of a complex variable. To achieve the goal, the following tasks were set:

1. Development of a physical and mathematical model of the process under complex asymmetric loading in conditions of single-area and two-area deformation zone during plastic processing of medium-thickness strips.

2. Investigation of the stress state, the process of loss of stability of strips of medium thickness.

3. Identification of patterns of changes in the stress state of the strip under conditions of the limiting deformation zone, the effects of reducing force loading at maximum compression.

The physical model of the process in the limiting zone is determined by different schemes of loading the deformation zone in the lagging and advancing zones. In the lag zone, there is a multidirectional action of the contact forces of normal pressure and friction along the deformation zone, and in the advance zone their co-directional action takes place. In the first case, tensile stresses appear, in the second case, longitudinal compressive stresses strengthen the support due to the contact friction force and normal pressure. As a result of the interaction of differently loaded zones in a single deformation zone, normal stresses from contact friction forces in the lagging zone decrease and increase in the advance zone [9, 10]. Based on experimental data, the influence of the shape factor on the distribution of normal and tangential stresses was demonstrated on Fig. 3 in [11]. At average values of the shape factor in the area of the lagging zone, the concavity of the contact normal stress diagram is clearly visible, which is explained by the influence of tensile stresses in this zone, and in the advance zone - the convexity of the diagram, due to additional force support.

A mathematical model must correspond to the presented physical model of the process. Considering the complexity of loading in a single deformation zone, it becomes necessary to use modern methods for solving problems of continuum mechanics, in relation to the plasticity theory [12, 13]. To enhance the result reliability, the solution of a closed problem of the plasticity theory using the method of functions argument of a complex variable was used [14].

EXPERIMENTAL

To analyze the stress state of the strip in processes with a near-limit deformation zone, only a part of the solution of a closed problem is used, without a deformation component. A solution for medium thicknesses is considered, since in this case it can be seen from Fig. 1 that the influence of tensile stresses in the lag zone is maximum. We have a statement of the problem in the form:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0; \quad \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0;$$

$$\left(\sigma_x - \sigma_y\right) + 4\tau_{xy}{}^2 = 4k^2 \qquad (1)$$

boundary conditions:

$$\tau_n = -\left[\frac{\sigma_x - \sigma_y}{2}\sin\left(2\phi\right) - \tau_{xy}\cos\left(2\phi\right)\right] \qquad (2)$$

Using the equilibrium equations and the plasticity condition Eq. (1), through the stress intensity T_i, it is possible to write a generalized equilibrium equation of the form [15]:

$$\frac{\partial^2 \tau_{xy}}{\partial x^2} - \frac{\partial^2 \tau_{xy}}{\partial y^2} = \pm 2\frac{\partial^2}{\partial x \partial y} T_i \sqrt{1 - \left(\frac{\tau_{xy}}{T_i}\right)^2} \qquad (3)$$

For linearization of boundary conditions and differential Eq. (3), trigonometric and fundamental substitution is used:

$$\frac{\tau_{xy}}{T_i} = sin\, A\Phi,$$

and

$$T_i = H_\sigma \, exp\, \theta, \qquad (4)$$

where $A\Phi$, Θ - the argument of the function that ultimately closes the solution; H_σ - the coordinate function responsible for the asymmetry of loading in the deformation zone.

Taking into account Eq. (4) and the functions of the complex variable, the tangential stress can be written [16]:

$$\begin{aligned} \tau_{xy} &= H_\sigma \exp\theta \sin \mathrm{A\Phi} = \\ &= H_\sigma \frac{\exp(\theta + i\mathrm{A\Phi}) - \exp\left(\theta - i\mathrm{A\Phi}\right)}{2i} \end{aligned} \qquad (5)$$

Substituting complex functions Eq. (5) into Eq. (3), we obtain:

$$\begin{aligned}
&\frac{1}{2i}\exp(\theta+i\mathrm{A\Phi})\cdot\Big\{\Big[(H_\sigma)_{xx}-(H_\sigma)_{yy}-2i(H_\sigma)_{xy}\Big]+2(H_\sigma)_x\Big[(\theta_x+\mathrm{A\Phi}_y)-\\
&-i(\theta_y-\mathrm{A\Phi}_x)\Big]-2(H_\sigma)_y\Big[i(\theta_x+\mathrm{A\Phi}_y)-(\theta_y-\mathrm{A\Phi}_x)\Big]+H_\sigma\Big[(\theta_{xx}-\theta_{yy}+2\mathrm{A\Phi}_{xy})+\\
&+i(\mathrm{A\Phi}_{xx}-\mathrm{A\Phi}_{yy}-2\theta_{xy})\Big]+H_\sigma\Big[(\theta_x+\mathrm{A\Phi}_y)-i(\theta_y-\mathrm{A\Phi}_x)\Big]^2\Big\}-\\
&-\frac{1}{2i}\exp(\theta-i\mathrm{A\Phi})\Big\{\Big[(H_\sigma)_{xx}-(H_\sigma)_{yy}+2i(H_\sigma)_{xy}\Big]+2(H_\sigma)_x\Big[(\theta_x+\mathrm{A\Phi}_y)+\\
&+i(\theta_y-\mathrm{A\Phi}_x)\Big]-2(H_\sigma)_y\Big[(\theta_y-\mathrm{A\Phi}_x)-i(\theta_x+\mathrm{A\Phi}_y)\Big]+H_\sigma\Big[(\theta_{xx}-\theta_{yy}+2\mathrm{A\Phi}_{xy})-\\
&-i(\mathrm{A\Phi}_{xx}-\mathrm{A\Phi}_{yy}-2\theta_{xy})\Big]+H_\sigma\Big[(\theta_x+\mathrm{A\Phi}_y)+i(\theta_y-\mathrm{A\Phi}_x)\Big]^2\Big\}=0
\end{aligned}\tag{6}$$

The same brackets appeared in the operators of both exponents of Eq. (6) during the transformation process $\Theta_x + A\Phi_y$ and $\Theta_y - A\Phi_x$. Moreover, these brackets create non-linearity, and it is quite reasonable to take them equal to zero. As a result, we come to the Cauchy-Riemann relations and the Laplace equation:

$$\Theta_x = -A\Phi_y,\ \Theta_y = A\Phi_x,\ \Theta_{xx} + \Theta_{yy} = 0,$$
$$A\Phi_{xx} + A\Phi_{yy} = 0 \tag{7}$$

Using the relations Eq. (7), the problem of identifying the argument of functions is eliminated Θ and $A\Phi$. After substitution Eq. (7) and simplifications, the differential Eq. (6) takes the form:

$$\begin{aligned}
&\frac{1}{2i}\exp(\theta+i\mathrm{A\Phi})\Big\{\Big[(H_\sigma)_{xx}-(H_\sigma)_{yy}-2i(H_\sigma)_{xy}\Big]-\\
&-\frac{1}{2i}\exp(\theta-i\mathrm{A\Phi})\Big\{\Big[(H_\sigma)_{xx}-(H_\sigma)_{yy}+2i(H_\sigma)_{xy}\Big]=0
\end{aligned}\tag{8}$$

One of the solutions of the system of differential Eq. (8) for a function H_σ can be the expression:

$$H_\sigma = \frac{C_0\left(\frac{l}{2}-x\right)+C_1\left(\frac{l}{2}+x\right)}{l}, \tag{9}$$

where C_0, C_1 – constants determining unequal stresses at the entrance and exit from the deformation zone; l – length of deformation zone.

Using dependencies Eq. (5) and Eq. (7), substituting them into the differential equations of equilibrium Eq. (1), after integration and substitution of boundary conditions, we have working formulas for determining the stress state of the strip under asymmetric loading in a single deformation zone:

$$\left.\begin{aligned}
\sigma_x &= -\frac{\frac{k_0}{\cos A\Phi_0}\left(\frac{l}{2}-x\right)exp(\theta-\theta_0)+\frac{k_1}{\cos A\Phi_1}\left(\frac{l}{2}+x\right)exp(\theta-\theta_1)}{l}\cos A\Phi+k_0,\\
\sigma_y &= -3\frac{\frac{k_0}{\cos A\Phi_0}\left(\frac{l}{2}-x\right)exp(\theta-\theta_0)+\frac{k_1}{\cos A\Phi_1}\left(\frac{l}{2}+x\right)exp(\theta-\theta_1)}{l}\cos A\Phi+k_0,
\end{aligned}\right\}\tag{10}$$

at

$\Theta_x = -A\Phi_y, \Theta_y = A\Phi_x.$

$$\theta_{xx} + \theta_{yy} = 0, \mathrm{A}\Phi_{xx} + \mathrm{A}\Phi_{yy} = 0$$

where $A\Phi_0$ and $A\Phi_1$, Θ_0 and Θ_1, k_0 and k_1, ξ_0 and ξ_1 - the meaning of the functions $A\Phi$ and Θ along the edges of the deformation zone; shear deformation resistance, coefficients that take into account the influence of backstop and tension during rolling at the entrance and exit from the deformation zone.

Having solved the Laplace equations, agreeing them with the Cauchy-Riemann conditions, we obtain the first argument function for the trigonometric dependence:

$$A\Phi' = AA_6'\left(\frac{l}{2}+x\right)y - AA_6''\left(\frac{l}{2}-x\right)y - 2\varphi =$$

$$= \boldsymbol{A}_6'\left[\left(x - X_0\right)+\left(\frac{l}{2}+X_0\right)\right]y +$$

$$+\boldsymbol{A}_6''\left[\left(x - X_0\right)-\left(\frac{l}{2}-X_0\right)\right]y - 2\varphi, \tag{11}$$

where $\varphi = (l - x)/R$ – inclination angle of the contact area; X_0 – position of the neutral section relative to the origin coordinates; A_6– constants determining the values of trigonometric functions at the edges of the deformation zone.

Considering the Cauchy-Riemann relations and the Laplace Eq. (7), the function Θ is determined.

$$\theta = -\frac{1}{2}\left(\mathrm{AA}_6' + \mathrm{AA}_6''\right)\left[\left(x + X_0\right)^2 - y^2\right] -$$

$$-\left(\mathrm{AA}_6' l_{i\hat{o}} - \mathrm{AA}_6'' l_{i\ddot{\imath}}\right)\left(x - X_0\right) \tag{12}$$

Using (11) and the boundary conditions, we obtain a neutral angle: to determine the lengths of the lag and advance zones:

$$\gamma = \frac{\alpha}{2}\frac{\mathrm{A}\Phi_1 - \alpha}{\left(\mathrm{A}\Phi_0 + 2\alpha\right)\left(1 - \frac{1}{2}\varepsilon\right)} \tag{13}$$

Expression Eq. (13) is necessary to determine the lengths of the lag and lead zones.

RESULTS AND DISCUSSION

The values of the relative contact stresses were calculated using formulas Eq. (10). To assess the reliability of the obtained result, it becomes necessary to compare the experimental data with the calculated values Eq. (10) [10].

Early the calculated stress values along the length of the deformation zone depending on the shape factor *l/h* was presented in [11], where after comparing the data in Figs. 2 and 3, it can be seen that with an increase in the shape factor, contact stresses increase, and the shape of stress plots changes. For large values of the shape factor (thin band), maximum specific pressures and process asymmetry are observed. The peak of normal and tangential contact stresses, with an increase in the shape factor, shifts towards the exit of the strip from the deformation zone. A complete coincidence of the result is observed with the form factor *l/h* = 1.03, despite the different presentation of the data. In this case, the loading asymmetry is completely absent, and the specific pressures take minimal values in terms of magnitude. The relative normal stress in the figure is represented by the ratio $\sigma_y / 2k$ tangential τ_{xy} / k.

The peculiarity of this comparison is the confirmation of the physical model, for which the effect of reducing contact stresses is enhanced during the rolling of medium-thickness strips in the lag zone. This is manifested by the deflection of the plots in the lag zone at the maximum values of the shape factor. Such deflections are explained by the presence of longitudinal tensile stresses, which, in combination with friction support stresses, cause an overall decrease in load [17].

Fig. 1 shows the stress distribution along the length of the deformation zone depending on the friction coefficient with a shape factor *l/h* = 8.61.

As can be seen from the Fig. 1, a change in the friction coefficient leads to a change in contact stresses in magnitude and in the distribution nature. Great interest has the deformation mode with a small friction coefficient (f = 0.05), when the maximum gripping ability of the rolls is reached. In this case, a single-area deformation zone is observed and at the same time, normal stresses noticeably decrease in magnitude and become less than the yield strength. This indicates the absence of plastic deformation, there is a slip of the strip in the rolls. The plot acquires a concave shape for normal stresses along the entire length of the deformation zone.

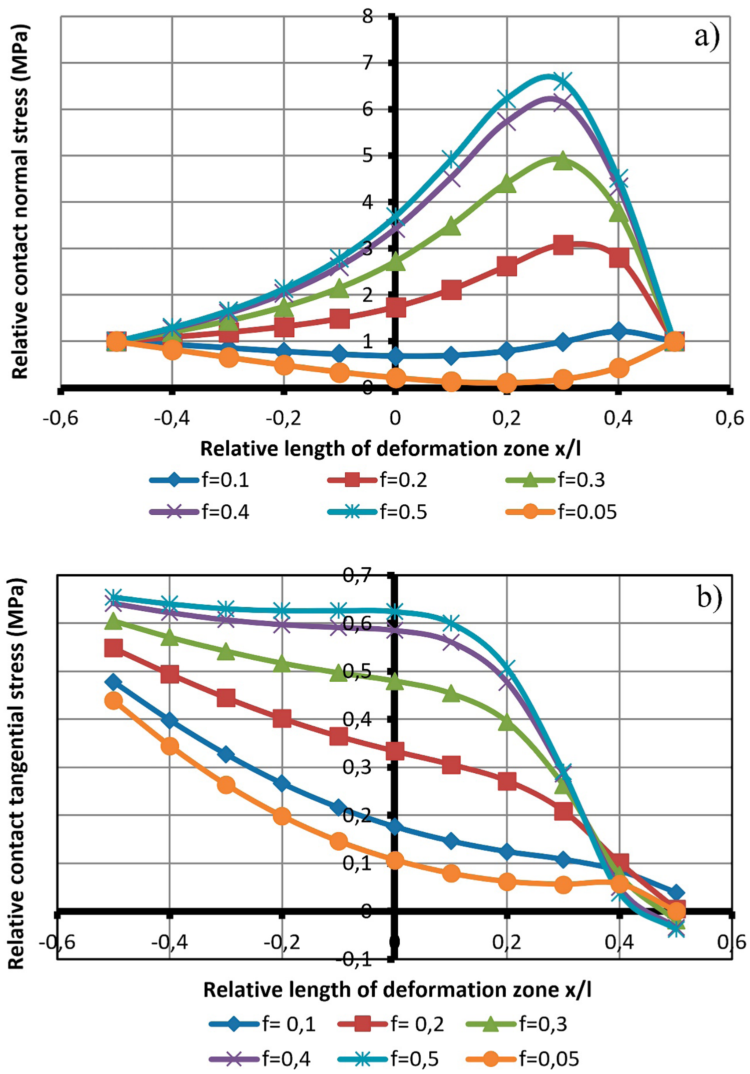


Fig. 1. Distribution of contact stresses along the length of the deformation zone depending on the friction coefficient at $l/h = 8.61$; $\alpha = 0.168$.

For a curve with a high friction coefficient (f = 0.1), concavity is observed only in the lag zone, loading is present in the advance zone. This type of distribution of contact stresses indicates a partial loss of stability. It should be emphasized that the mathematical model of the process reacts to a single-area deformation zone, confirming the experimental data that the rolling process can be implemented even with negative advance [5].

The revealed feature or the effect of strips rolling process of medium thickness is of not only theoretical but also practical interest, since a mutually opposite pattern has been revealed - an increase in compression with a weakening of the stress state in the zones of accessibility of the limiting deformation zone.

Fig. 2 shows the distribution of contact stresses at different compressions with a friction coefficient 0.4. In this case, an increase in the gripping angle corresponds to an increase in the maximum normal contact stresses. Tangential stresses also react in the presence of a two-area zone for all deformation modes. In lagging areas, contact stresses decrease faster with increasing compression than for processes with less compression.

Contact stresses react approximately the same way to changes in the capture angles at a friction coefficient 0.3. Large compressions correspond to large normal and tangential stresses at a two-area deformation zone. As in

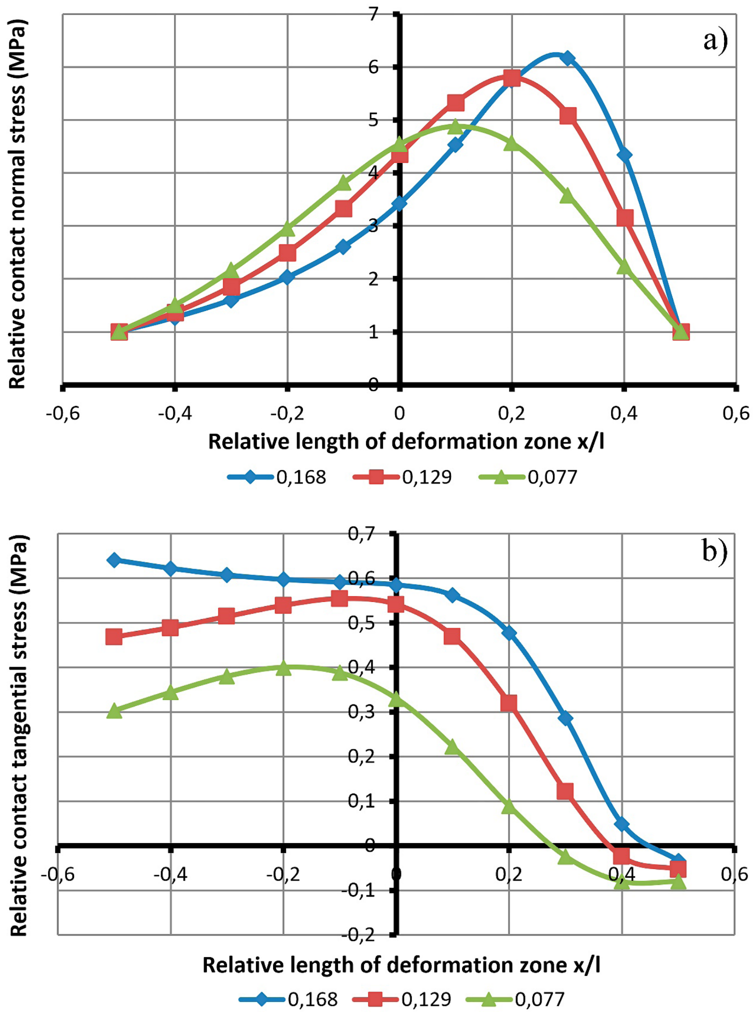
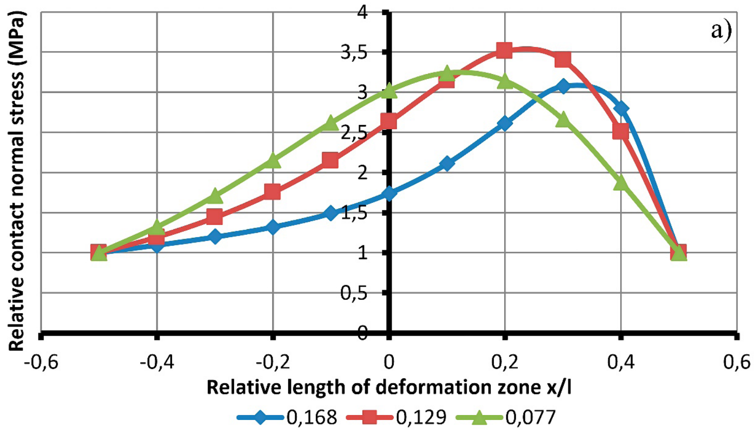


Fig. 2. Distribution of contact stresses along the length of the deformation zone depending on the capture angle (α = 0.168; α = 0.129; α = 0.077) with a friction coefficient 0.4 at *l/h* = 8.61.

the previous case (f = 0.4), the advance zone decreases with increasing compression [18, 19]. This shows that the mathematical model reacts to the stability of the process under certain parameters of plastic deformation. It should be emphasized that such factors of stability of the process may be the presence of two-area deformation zone, which is explained by the complex interaction of differently loaded lag and advance zones in a single zone of plastic flow.

With a further decrease in the friction coefficient (f = 0.2), the reaction of the stressed state of the medium to the gradual approach of the process to the limit, i.e., the loss of its stability, was considered. Fig. 3 shows the diagrams of the distribution of contact stresses at a friction coefficient 0.2. In this case, the capture angles are as close as possible to the friction coefficient. We have two two-area deformation zones and one single-area zone, which is allowed by the condition of negative advance [5].

In the lag zone, the tangential stress curves at the entrance to the deformation zone indicated the stresses in magnitude in accordance with the compressions. In the advance zone at the exit from the deformation zone, this condition is also observed, however, with maximum compression, a single-area deformation zone is realized, with zero advance. This is a prerequisite for the lane slipping process. The curves of normal stresses were distributed differently. There has been a fundamental restructuring of the stress state in the reach area of the limiting deformation zone. According to the force



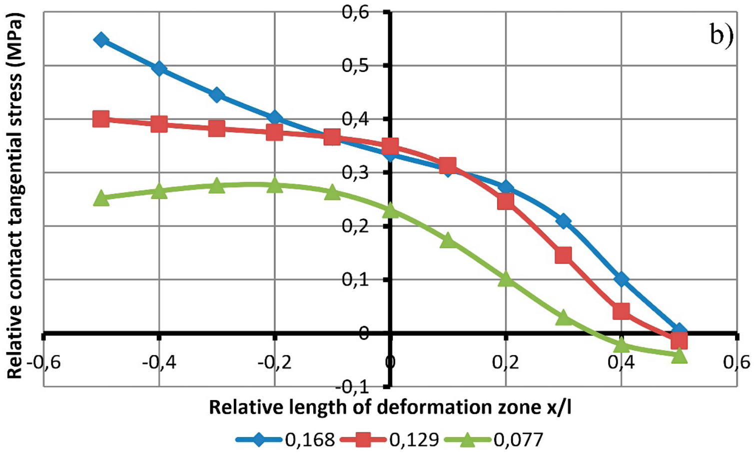


Fig. 3. Distribution of contact stresses along the length of the deformation zone depending on the capture angle (α = 0.168; α = 0.129 and α = 0.077) with a friction coefficient 0.2, at *l/h* = 8.61.

loading, the maximum compression corresponds to an almost equal loading with a minimum capture angle. The maximum loading is a process with intermediate compression. There is a violation of the correspondence between the capture angles and stresses in the deformation zone. The maximum compression corresponds to the minimum cantate stresses. In the lagging areas, the discrepancy between the power load and the capture angles increased. At the same time, the loss of stability of the rolling process has not yet been observed.

The mathematical model allows to visually consider the process in which the friction coefficient becomes less than one of the capture angles (Fig. 4). In this case, the process partially loses stability. The question arises what happens to the stressed state of the plastic medium in conditions of complete or partial loss of stability of the process.

There are two single-area deformation zones and one two-area zone, which corresponds to minimal compression. At the entrance to the deformation zone the tangential stresses were distributed in accordance with their compressions, as before. At the point of exit from the deformation zone, such a correspondence was preserved, but only at the exit point. The distribution of normal stresses reacts to the increased influence of the limiting deformation zone. The changes in power characteristics have intensified. The maximum compression fell out of consideration, because There is no plastic deformation in most of the deformation zone, there is a partial slip. The highest stresses correspond

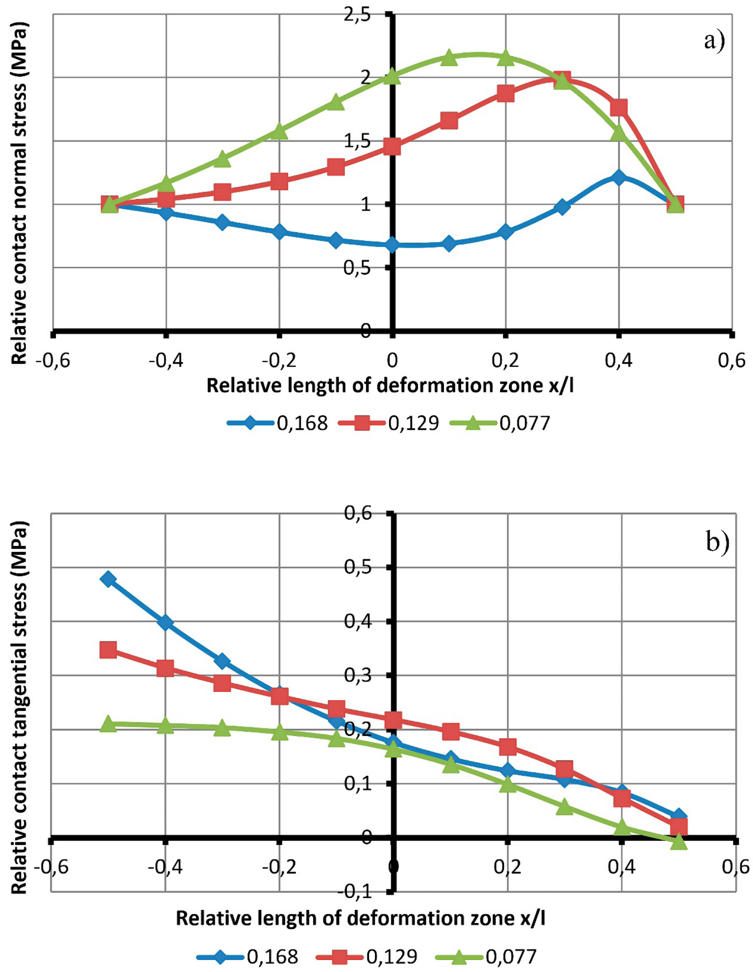


Fig. 4. Distribution of contact stresses along the length of the deformation zone depending on the capture angle (α = 0.168; α = 0.129; α = 0.077) with a friction coefficient 0.1, at *l/h* = 8.61.

to the minimum compression. The intermediate capture angle is characterized by lower stresses, with a stable process in the single-area deformation zone. Partial loss of stability is determined by the fact that for maximum compression, stress reduction in the lag zone below the plasticity limit does not occur along the entire length of the deformation zone. There is a plastic contact in the advance zone.

Fig. 5 shows the force parameters of the limiting deformation zone, in particular, two processes with a single-area deformation zone and one two-area zone are presented. For tangential stresses at the entrance and exit from the deformation zone, there is some correspondence in magnitude with the capture angles. There is no such correspondence in the central part of the hearth, but there is a loss of stability of the process along the entire length of the deformation zone. For tangential stresses, this is a single-area deformation zone, for normal stresses, the concavity of the contact stress distribution plot along the entire length of the deformation zone. Thus, the mathematical model clearly characterizes the stress state when the strip is slipping under conditions of maximum capture angle.

It should be noted that in the reach area of limiting deformation zone, the effect of plastic shaping has been revealed, which determines a contradictory pattern. Reduction of the force load with an increase in the deformation load.

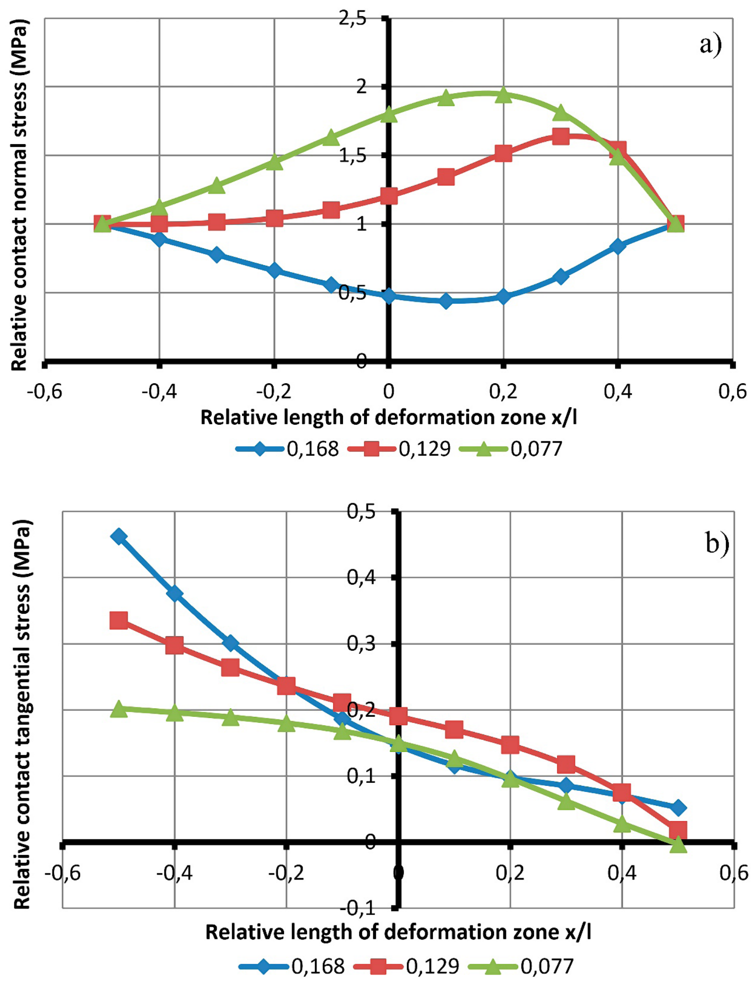


Fig. 5. Distribution of contact stresses along the length of the deformation zone depending on the capture angle (α = 0.168; α = 0.129; α = 0.077) with a friction coefficient 0.08, at *l/h* = 8.61.

CONCLUSIONS

Based on the results of the conducted research, a physical-mathematical model of the process under complex asymmetric loading under conditions of single-area and two-area deformation zones during plastic processing of medium-thickness strips was developed. During the study of the stress state by the visual approach of the plastic medium in conditions of limit and near-limit deformation zones, interaction patterns of differently loaded areas of a single deformation zone on the force parameters of rolling were revealed. During the variation of the main values of the friction coefficient and capture angle, patterns of changes in the stress state of the medium under conditions of reaching the zone of extreme deformation, the effects of plastic shaping determined by a decrease in contact stresses under conditions of increasing deformation load were revealed.

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